Wave-particle complementarity: teaching quantum physics with a Virtual Mach-Zehnder Interferometer

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Abstract
We present the results of a research conducted with a computational simulation of the Mach-Zehnder Interferometer, which allows one to perform several experiments involving single photons. In this study, we have investigated how undergraduate students (pre-service teachers) adopt discursive strategies to understand the concept of wave-particle complementarity when they work collaboratively, paying special attention on the software, used as a mediational tool in these discursive interactions. Based on the Vygotsky’s ideas on learning and development and assuming that language is the main cultural human mediational tool, the sociolinguistics of Mikhail Bakhtin has been adopted to increment analysis on the students' verbal interactions, which revealed interesting strategies adopted to understand the concepts involved.

Keywords
Teacher Education, Complementarity Principle, Virtual Mach-Zehnder Interferometer

Introduction
Researches on physics education have been conducted in the twentieth century in Brazil, pointing the need and feasibility of insertion of modern physics topics in high school level. Since the first studies related to the teaching of this subject in Brazil, conducted in mid-1990s, several possibilities to promote the insertion of this topics have been proposed (Pietrocola, 2005). Considering the period up to the present day, it can be said that there is still much to be done to put forward the consolidation of Modern and Contemporary Physics (MCP) in high school. In a country with continental dimensions like Brazil, national assessments are performed and the results reveal large learning disparities between different regions and schools (public schools have worse results if compared to private schools). There are frequently complaints about deficient conditions of schools (mainly the public ones), the low amount of physics classes and teachers’ lack of self-confidence to teach MCP in their schools.
In fact, the twentieth century physics is taught in some schools, but this does not always happen. When it happens, time is usually dedicated to explore topics such as the special theory of relativity, blackbody radiation and atomic models, and some other physics topics of early twentieth century. Fundamental aspects of quantum physics are simply left out. On the other hand, one can also question about the way physics teachers are being taught in their undergraduate courses. It is common to listen teachers say that their physics courses were too much centered on mathematical procedures, most often with a considerable degree of complexity. Moreover, these approaches often ignore epistemological and ontological aspects of quantum objects, which could be helpful to conceal a more conceptual approach. The rejection feeling on many physics teachers regarding to implementation of quantum physics in the high school curriculum, may be caused due to the image that was created in their formative experience, reminds unfamiliarity with mathematical formalism and other serious difficulties (Monteiro et al., 2009). Considering these difficulties, we opted to take a stance towards improvement of the initial training of physics teachers, since among the difficulties alleged by the teachers is lack of self-confidence to teach quantum physics. More familiarity with this topic can provide more confidence to teachers, leading them to feel more comfortable to teach fundamental concepts of quantum theory in high school.

Several researchers have developed studies on the teaching of quantum physics in undergraduate courses in the last decade. Among other issues, the researchs are aimed to investigate students’ difficulties on their understanding of the core of this theory, to conceive alternative teaching methodologies and/or to develop teaching resources. These researches include development of experimental activities (Galvez et al., 2005), use of interactive tutorials (Singh, 2008), teaching of quantum interference with Virtual Mach-Zehnder Interferometer (VMZI) (Pereira et al., 2009), tutorial for simulating the Stern-Gerlach experiment (Zhu and Singh, 2011), a collection of interactive animations of quantum physics and proposal of a new curriculum to teach it (Kohnle et al., 2012; Kohnle et al., 2014). Use of experimental activities and computational simulations as didactical resources are highlighted as ways to promote conceptual approaches to some quantum phenomena. One of these resources is the VMZI, a computational simulation of the Mach-Zehnder Interferometer, wherewith one can perform several experiments involving single photons (e.g. quantum interference, non-demolition detection, photon polarization, etc.).

**The Virtual Mach-Zehnder Interferometer and the wave-particle complementarity**

This software, developed originally in 2004 by our research group, was remodeled in 2012-2013 to account several additional phenomena. The original version was used in teaching activities on introductory quantum physics undergraduate disciplines, as part of research projects conceived by our group. With this newer version, our intent is to take a step into more advanced topics, allowing more parameter configurations. The figure 1 shows a simplified diagram of Mach-Zehnder interferometer, along with examples of interference patterns. In figure 2 is depicted the layout of the current software version. This diagram represents an overview of the interferometer. There are two input ports (I and II) and two output ports (1 and 2). A monochromatic photon source is placed at input port I and screens at output port 2. To avoid unnecessary physical complications, we consider that the beam splitters (BS1 and BS2) are cubic and symmetric (Zeilinger, 1981; Hamilton, 2000; Holbrow et al., 2002). Also, we
consider that each path, A or B, has equal lengths. Among other possibilities, it’s possible to change values of reflection ($R_1$ and $R_2$) and, thus, of transmission coefficients ($T_1 = 1 - R_1$ and $T_2 = 1 - R_2$) in both beam splitters.

![Figure 1. Mach-Zehnder interferometer with symmetric unbalanced cubic beam splitters BS$_1$ and BS$_2$, with reflection (transmission) coefficient $R_1$ ($T_1 = 1 - R_1$) and $R_2$ ($T_2 = 1 - R_2$), respectively. Two input ports (I and II) and two output ports (1 and 2) are available. A light source is placed at input port I and screens are placed at the output ports 1 and 2 to observe the interference patterns. Examples of interference patterns is shown at right, for classical and quantum pictures (in the case of a balanced beam splitters, i.e. $R_1 = R_2 = T_1 = T_2 = 0.5$). The vertical line at the centre of each screen acts as an aid to visualize that interference pattern on screen 1 is inverted related to pattern on screen 2.](image)

For values other than 0.5 for reflection (and transmission) coefficient, the beam splitters become *unbalanced* and photon path information becomes available (in this case, it is possible to get knowledge about the path, A or B, associated to the photon inside the interferometer, decreasing the contrast of interference pattern). The possibility of choosing the value of reflection coefficient for each beam-splitter opens the possibility to explore and investigate students responses to teaching activities focused on the quantitative wave-particle complementarity (Li et al., 2012). In Mach-Zehnder interferometer, the complementarity between the wavelike behavior and particle-like behavior can be stated, according to Auletta et al. (2009, p. 19), as follows: “the complete knowledge about the path is not compatible with the presence of interference”. Here, the word “path” should be understood as being associated with a translational state of the photon in the interferometer, as defined by Dirac (1958, p. 7). Complementarity is one of the main thesis of Copenhagen Interpretation of quantum Physics. The wave particle-complementarity is the most common kind of complementarity that appears on physics textbooks. Thus, we made the option of address the wave-particle duality in the light of Copenhagen Interpretation. Despite no agreement exists on which is the best interpretation of quantum physics, Copenhagen Interpretation seems to be the one which attracts more scientists on this research field, followed by the Many-Worlds Interpretation (Schlosshauer et al., 2013). As stated by Muyneck (2004, pp. xxi, on preface), without interpretations, quantum physics would be only mathematics. To implement meaning and “make it physics”, an interpretation is needed “in the sense of a mapping of entities of the mathematical formalism into reality is indispensable”.

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**Figure 1.** Mach-Zehnder interferometer with symmetric unbalanced cubic beam splitters BS$_1$ and BS$_2$, with reflection (transmission) coefficient $R_1$ ($T_1 = 1 - R_1$) and $R_2$ ($T_2 = 1 - R_2$), respectively. Two input ports (I and II) and two output ports (1 and 2) are available. A light source is placed at input port I and screens are placed at the output ports 1 and 2 to observe the interference patterns. Examples of interference patterns is shown at right, for classical and quantum pictures (in the case of a balanced beam splitters, i.e. $R_1 = R_2 = T_1 = T_2 = 0.5$). The vertical line at the centre of each screen acts as an aid to visualize that interference pattern on screen 1 is inverted related to pattern on screen 2.
A thought experiment to observe intermediate quantum interference phenomena was first proposed by Wootters and Zurek (1979) and, since then, complementarity principle has gained wide attention. Among others, Englert (1996) obtained a general duality relation between visibility of interference pattern and path distinguishability in the context of two-way interferometers. These two quantities are very important to quantify particle-like and wavelike behavior of the quantum object, especially in intermediate situations where the visibility is less than one. The path distinguishability can be understood as a physical parameter that measures the degree of particlelike character of the quantum object (Greenberger and Yasin, 1988). It is related also to the maximum probability of making a correct choice about the path associated to a photon that produced a particular mark in a screen. In the Mach-Zehnder interferometer, it can be shown that the distinguishability in each screen is given by

\[ D_1 = \frac{R_2 (1 - R_1 - R_2) (1 - R_2)}{[R_2 (1 - R_2) + R_2 (1 - R_2)]} \] on screen 1 and

\[ D_2 = \frac{R_1 R_2 - (1 - R_1) (1 - R_2)}{[R_1 R_2 + (1 - R_1)(1 - R_2)]} \] on screen 2. Moreover, it can be shown that the maximum probability of making the right choice about the path in the interferometer associated to the photon that produced a mark on the i-th screen is

\[ P^i_{\text{max}} = \frac{1 + D_i}{2}, \] in which \( D_i \) is distinguishability in detector or screen \( i \) \( (i = 1, 2) \), and

\[ 0 \leq D_i \leq 1. \]

Figure 2. Capture of the software interface showing one of its possible layouts (1). In (2), we show how the classical (option Laser) or quantum (option Single Photons) pictures of interferometer can be selected – dropdown options are in detail in (3). Also in widget (2), input values of parameters (e.g. \( R_1 \) and \( R_2 \)) can be entered. Numerical values of interference visibility in each screen appear in the bottom of this widget. In (4) are shown all possible photon counters, providing theoretical predictions and results of simulations.

The visibility, or contrast of the interference pattern, can be thought as a physical parameter that quantifies the degree of wavelike character of quantum object (Greenberger and Yasin, 1988). It can be shown that the visibilities in VMZI on each screen is given by

\[ V_1 = 2\sqrt{(R_1 R_2 (1 - R_1)(1 - R_2)) / (R_1 (1 - R_1) + R_1 (1 - R_2))} \] on screen 1, and

\[ V_2 = 2\sqrt{(R_1 R_2 (1 - R_1)(1 - R_2)) / (R_1 R_2 + (1 - R_1)(1 - R_2))} \] on screen 2. Analogous to
distinguishability, \(0 \leq \mathcal{D}_t \leq 1\). The complementarity between wavelike and particle-like behaviors of the quantum object in VMZI is quantitatively described by \(\mathcal{V}_t^2 + \mathcal{D}_t^2 = 1\), on each screen. If we consider the maximum distinguishability (equal to 1), it means that there is complete information about the path and this implies zero visibility (no interference or full particle-like behavior). In the complementary situation, there is no path information available or, in other words, distinguishability is zero, implying that the visibility will be maximum (interference pattern with maximum contrast, indicating full wavelike behavior). If we consider only these two extreme cases, path distinguishability, or ability to take a “trajectory” inside the interferometer (a particle-like behavior) and ability to produce interference pattern (wavelike behavior) are complementary or mutually exclusive behaviors. It is not possible to configure an experiment so that all of the quantum objects exhibit these two behaviors at once. However, the most interesting cases to be studied are exactly those where visibility and distinguishability values are smaller than unity and satisfy the quantitative relation \(\mathcal{V}_t^2 + \mathcal{D}_t^2 = 1 (t = 1, 2)\). For the cases in which path information is partial and the interference pattern has reduced visibility, the wavelike and particle-like behaviors appear simultaneously. This is a generalization of the original conception of Bohr's complementarity between the corpuscular and wavelike nature, present in most physics textbooks.

**Theoretical framework**

In this paper we investigated ways in which undergraduate students (pre-service teachers) can understand the concept of wave-particle complementarity, paying special attention on how the software, as a mediational tool, acts in order to promote creation of the Zone of Proximal Development (ZPD) when they collaboratively work in pairs (Rio and Álvarez, 2007). Coherently with this aim, the theoretical framework adopted in this study is the Vygotsky’s mediation theory, revisited by James Wertsch and other scholars. This theory assumes *mediated action* as the most important aspect in Vygotsky’s theory: any social action performed by humans uses mediational tools (Wertsch, 1993). The VMZI is a mediational tool that allows interaction between the students and simulated physical phenomena. Furthermore, language is the main human cultural mediational tool used in social contexts – discursive interactions between pre-service teachers will be our main focus of analysis (Vygotsky, 1978). Assuming that language is the main human cultural mediational tool, the sociolinguistics of Mikhail Bakhtin has been adopted to perform discourse analysis on the students’ interactions (speech and discursive interactions), pointing the relationship between their discursive exchanges and the organization of their actions during the teaching activities (Bakhtin, 1997). According to Bakhtin's theory of enunciation, the speeches are not associated only with one voice but at least two voices. This process, which one voice is not the only responsible for the creation of an utterance, but in fact the words are part of another discourse, Wertsch (1993) calls interanimation. This multiplicity of voices that permeate the discourse indicates that it is not strictly restricted to the context in which it occurred. The speech carries relationships, sometimes almost imperceptible, with ideologies, institutional norms, culture, aspects that may be far away, spatially or temporally, from the context in which the speech occurs.

**Research methodology and data analysis**

The teaching activities were conducted with students organized in pairs. Each of these pairs of students received an experimental guide with questions to be answered during the simulation on
VMZI. The dialogues between students during classroom were recorded in audio and video. In this paper, we present some discursive exchanges of a pair of students (S7 and S8) and the analysis that we perform is mainly focused on two central bakhtinian concepts: counterwords and voices.

**Figure 3.** Upper: Mach-Zehnder interferometer with interference patterns obtained for $R_1 = 0.20$ and $R_2 = 0.10$. Lower: Mach-Zehnder interferometer with interference patterns obtained for $R_1 = 0.95$ and $R_2 = 0.05$.

**Dialogues in classroom**

After some introductory classes, the teacher presents situations on VMZI. One of these situations is depicted in upper part of figure 3, mediated by a text guide, in which students are asked to explain the interference patterns on the screens when the reflection probabilities of the beam splitters are $R_1 = 0.20$ and $R_2 = 0.10$, as shown in upper part of figure 3.

1. S8: Here [screen 1] the visibility is greater than in the other [screen 2]. The distinguishability is the opposite. It is larger in the screen 2. Increase the visibility leads to decrease of distinguishability.
2. S7: Yes.
3. S8: Increased visibility makes the distinguishability decreases.
4. S7. No. *The observer determines the distinguishability* before, at the beginning [of experiment].
6. S7: Is the observer who determines it. Do you remember that the teacher said? *If you put a detector, you change the visibility.*
7. S8: (...) Okay. But, how do I do it? How do I choose the distinguishability?
8. S7: Yeah, more or less. Let me note here. *It has interference. It is wavelike behavior.*
9. S8: (...) Oh, here [screen 1] it is difficult to know the path.
10. S7: Yes, it is. The visibility is 0.92.

The student S8 shows trouble on understanding quantitative wave-particle complementarity. Although he has correctly contrasted the decrease of visibility when the distinguishability increases, it is clear that he does not understand S7’s utterance on line 4 (in this specific situation, the distinguishability has been defined on preparation of the experiment, choosing the parameters $R_1$ and $R_2$ of the beam splitters). Is S7 who alerts S8 about this possibility (lines 4 and 6) as well about the more prominent wavelike of photons that hit screen 1 (line 8). Here, S7 assumes the role of more capable peer, showing more mastering on semiotic reasoning about the phenomena and helping to open the ZPD between him and S8. Student S7 interanimates his voice with a voice closest to experts’ voices, since he has some fluency with terms like observer or wavelike behavior and answering S8 (line 9) correctly about the lack of path information on screen 1, evoking the concept of visibility (line 10). Possibly textbooks or a deepest understanding of teacher introductory classes inspired him. His voice interanimates also with S8’s voice, taking an actively responsive stand on dialogue, conducting it. The teacher does not even take part on this dialogue and the process was carried out only guided by S7 with clear mediation of the VMZI, as psychological and technical tool. The psychological aspect refers to semiotic dimension, with its available semiotic resources, most of pictorial nature, like the interference patterns or the 3D representation of the interferometer and its components as a virtual microworld. These semiotical resources allow students to reason about and interpret the results of their actions on the software in the light of quantum physics concepts like distinguishability, for example. The technical aspect refers to the fact that VMZI allows concrete actions – e.g., choosing different parameters for both beam splitters – on a virtual world, where the interferometer is the main virtual component.

In the sequence of activity, the students are asked to change the reflection coefficients for $R_1 = 0.95$ and $R_2 = 0.05$, as shown in the lower part of figure 3. Again, they were asked about distinguishability of the path and formation of the pattern of detections.

11. S8: I don’t believe. (...) Now it’s all mixed.
12. S7: (...) It is very erased here [screen 2]. But the pattern is more clear than other.
13. S8: It is erased because has only 18 percent of the photons here. What did he ask?
14. S7: It's that thing again. It has interference on screen 2 and thus the distinguishability is zero.
15. S8: What is the question?
16. S7: He asked if it is wave or particle. You have to talk about the information available.
17. S8: (...) On screen 2 we have wavelike behavior, then disappear the interference. I think it is strange, because photons are particles.
19. S8: ... Yeah, I guess that's it. Some photons are particles and other photons are waves.
20. S7: Yeah, but you have to talk about the information.
21. S8: What information?
22. S7: The information about the path of the photons.
23. S8: (...) 95 percent [of photons] are reflected on the first beam-splitter and goes through this path [path A]. Do you agree with this?
24. S7: Yes, because they suffer reflection. The same in the second beam-splitter. This is it. Most photons hit screen 1. We know [much more about] the path of the photons detected here.
25. S8: ... So, let's write down this sentence.
26. S7: Okay, this is the partial information.
27. S8: We have no sure for every photon.
28. S7: Is not it.
29. S8: Half of photons are wave and other half is particle.
30. S7: No, no, no. You are confuse. We have partial information available for all photons, but we do not know exactly where the photons will be detected. We know only the probability.
31. S8: This depends if they are wave or particle?
32. S7: No. Yes. It's almost this.

In this interaction, again S7 plays the role of more capable peer in several utterances (lines 14, 16, 20, 22, 24, 26, 28, 30). This role is important not only for S8, but for himself. He develops linguistic (discursive) strategies not only aiming convinces S8, but to reason about the complex situations involved here. Again, he interanimates his voice with voices close to expert voices, embedded with linguistic maturity to ground his reasoning about the situation in terms of path information, explaining the interference patterns supported by the concept of path distinguishability and correctly associating it with wavelike or particlelike behavior (lines 14, 18, 24 and 30). Student S8 seems to adopt a classical reasoning in some utterances, considering photons as particles like bullets, showing difficulty to understand interference on screen 2 (line 17). After discussion with S7, he seems to be more comfortable with the idea of photon dual behavior on that screen, although he still seems to consider this issue in a naïve way.

Conclusions

The analysis presented in this paper reveals some interesting results that can help teachers to identify problems on the students understanding of wave-particle complementarity of photons and therefore to adopt strategies that promote their learning. The speech of student S8 reveals that there is an underlying tendency to attribute particlelike behavior to the photons, even in situations where wavelike behavior is evident. Although the particlelike behavior attributed to the photons doesn't indicate that student adopted truly a 'corpuscular interpretation', to fall back on to this way to represent the photons leads to difficulties in the understanding of the quantitative complementarity principle. In discursive interactions like these presented here, the role of more capable peer is evidenced as very important not only for the other element of the pair (the less capable peer), but for himself. Is the dialogical interaction with the other that creates a ZPD and leads both to better understand the concepts involved by negotiating meanings and to organize their actions along the teaching activities. The mediational role of IVMZ is clearly important on these processes. Furthermore, studying the discursive strategies used by pre-service teachers to understand the concepts addressed here, it can help to improve the teaching of quantum physics in pre-service teacher formation programs. The conclusions of this study lead us also to new questions about the learning of others fundamental concepts of quantum physics, such as nonlocality. What can IVMZ reveal in relation to the process of
student’s understanding even more counterintuitive situations, such as quantum entanglement? How does particlelike behavior attributed to photon can interfere with the understanding of quantum entanglement? These questions will be a continuation of this work, in which the software will be updated to allow the simulation with pairs of photons in entangled polarization states.

Acknowledgment

The authors of this paper thanks the Brazilian National Research Committee (CNPq) for partial financial support.

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