What is light?  
From optics to quantum physics  
through the sum over paths approach

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Abstract
In this work we propose a learning sequence on the nature of light, starting from wave optics and leading to quantum physics. Feedback from previous in-service teacher courses convinced us that a revision of the traditional approach to wave optics can favour a gradual and effective approach to quantum concepts. Moving from an established research line on the educational use of the Feynman sum over paths method, we make an effort of clarifying several conceptual and epistemological aspects of quantum theory by presenting modern experiments, mainly from the field of quantum optics, from the point of view of Feynman paths. In addition, we implemented interactive simulations allowing a visualization of Feynman’s model in selected physical situations using the widely praised, open source software GeoGebra, and coupled them to a solid experimental base using low-cost equipment and the Tracker software, also open source. We tested our approach in a course for pre-service teachers, obtaining encouraging results.

Keywords
Teaching quantum physics, Feynman paths, interactive simulations, interferometry.

Introduction
The history of the answers to the question “what is light?” runs as a fascinating thread intertwined with the history of physics as a whole. During their high school curriculum in science-oriented programs, many students encounter three different models of light, at different level of sophistication: light as a ray (geometrical optics), light as a wave (wave optics) and finally, light as a photon (in introductory quantum physics). Often no attempt is made to connect these models, and to provide a unified picture, and a global answer to the question of the nature of light. However, Feynman’s sum over paths approach, which has a consolidated tradition in the teaching of quantum physics (Taylor et al., 1998; Cuppari et al., 1997, de los Ángeles Fanaro et al., 2012; Dobson, 2000; Ogborn & Whitehouse, 2000) allows to build a strong connection between the wave phenomenology of light and the properties of the photon; also, it provides a particularly clear explanation of the emergence of geometrical optics in the limit of small wavelengths with respect to other length scales of the system.

With respect to existing proposals, our work tries to address more in depth conceptual and epistemological themes which are peculiar to quantum physics by treating, in the language of Feynman paths, modern interferometry experiments in which non classical features are clearly focused. Themes such as the impossibility to attribute a trajectory to particles, the role of measurement and wave function collapse, the uncertainty principle, the consequences of indistinguishability of identical particles, can be formulated in particularly clear terms when
seen from the perspective of the sum over paths approach. Modern interferometry experiments can also serve as a natural introduction to current, information based foundational views, which the language of Feynman paths is particularly suited for expressing. For example, in the sum over paths perspective, “wave particle duality” is simply a consequence of the reduction of possible paths for the system, due to acquired information. With the loss of possible paths, interference is lost, and so is the wave character of the quantum object. Thus the Feynman model allows to provide a verbal explanation (rationalization) of duality, as seen for example in two slit interference.

For the purpose of letting students familiarize with Feynman’s model and explore its features in relevant physical settings, we developed interactive simulations using the software GeoGebra, which is widely used and praised in the community of teachers of mathematics and physics (Hohenwarter et al. 2009). GeoGebra offers the possibility of a high level of interactivity and manipulation through the use of sliders and boxes; it allows to build an intuitive connection between two different dynamical representations by seeing them paired in adjacent workspace windows; and finally, permits to save one’s work in common open formats.

Organization of the sequence

The sequence starts from the phenomenology of wave optics, as observed in the classical experiments of interference and diffraction. Experiments are performed and discussed using Tracker as a tool to produce a quantitative analysis of interference fringes. The method of paths and phasors is introduced as a convenient way of representing wave phenomena, and its results, derived from simulations and direct calculations, are compared to experiment.

The next step is the discussion of experiments leading to the necessity of a photon model: photoelectric effect, two slits interference with individual photons, and the Grangier experiment on photon indivisibility. Each experiment is discussed in its own significance, and the model of the photon following all possible paths from source to detector is derived as a logical consequence.

A central part of the sequence is devoted to discussing conceptual and epistemological themes through the analysis of modern quantum optics experiments in the language of Feynman paths. Experiments discussed include the single photon Mach-Zehnder, Zhou-Wang-Mandel, and Hong-Ou-Mandel experiments. One of our reasons for departing from an historical approach is that for some concepts the current interpretation is significantly different from what it was in the first years after the formalization of quantum theory, having evolved in response to new evidence and emerging problems. A paradigmatic example is the uncertainty principle, which is today no longer thought as being the result of a perturbation of the system due to measurement (as in the Heisenberg microscope thought experiment), but as an intrinsic limitation in the available information about a quantum system.

In the final part of the sequence we provide students with a connection to the model of geometrical optics, by showing through simulations that, in the sum over paths approach, the “least time” path becomes increasingly dominant as the ratio between the wavelength of the photon and other characteristic length scales of the system gets smaller. Thus we “close the
circle”, with a photon model of light which can explain quantum phenomena, and account for both the alternate models which students were accustomed to.

Key turning points

Interference in wave optics and the Huygens principle

The sum over paths method can initially be seen as a convenient way for describing interference phenomena in a classical wave perspective. In our presentation the method is introduced through a discussion of the Huygens principle, whose idea of a wave producing new wave sources at all points in space, has well known analogies with the path integral formulation. (Ogborn & Whitehouse, 2000) Students learn to compute the value of the amplitude of a monochromatic wave at a given point P in space by summing the amplitude vectors resulting from all possible “optical paths” that an elementary disturbance composing the wave could have followed to reach P from the original source S. The source-to-detector (Dobson, 2000) philosophy is central in the sum over paths perspective, and is often also an implicit assumption in the usual treatment of interference phenomena in wave optics.
Figure 1. (A) Animation illustrating the connection between the Huygens principle and the idea of using all possible optical paths from source to detector. (B) Two slit interference pattern with superimposed Tracker analysis of the light intensity. (C) GeoGebra simulation of the two slit interference.

The photon concept and the Grangier experiment

In introducing the photon idea we use experimental evidence gathered at different times, in the unifying perspective of providing a logically convincing, unambiguous construction of the idea of photon and of its fundamental properties. In particular, we discuss the following experiments:

- The photoelectric effect, as basic evidence of granularity of light interacting with matter, and allowing to discuss the quantization relation $E = h\nu$.
- Two slit interference experiments with single photons (Jacques et al., 2005) introducing the probabilistic interpretation.
- The Grangier et al. 1986 experiment (Grangier et al., 1986) on photon indivisibility.

Our message to students can be summarized as follows: when light is thought of as a wave, it naturally has the property of being distributed in space so that, for example, it can pass through both of the two slits in the Young experiment, producing interference. But how can the phenomenon be explained if light is made of photons, and interference persists even if the intensity of light is so low that one at a time is emitted by the source? The Feynman idea of the photon following all possible paths is then presented as a logically consistent answer. In this perspective, the Grangier experiment (Figure 2) plays a very important role. In fact, students who are confronted with the phenomenon of single photon two slits interference, and the model of the photon following all possible paths from source to detector may form the hybrid (synthetic) conception of each photon splitting in two at the slits. The discussion of the Grangier experiment on photon indivisibility at this point is intended to address the specific difficulty at the precise moment when it may show up.

Figure 2. Setup for the Grangier experiment on photon indivisibility
Single slit diffraction: introducing the uncertainty principle

The phenomenon of single slit diffraction with a variable slit width is discussed, both experimentally and through the use of a simulation (figure 3), as a simple but effective introduction to the uncertainty principle (Johansson & Milstead, 2008). The diffraction pattern can be easily related to the momentum probability distribution of the diffracted photon, and as the slit width is varied in a simulation, an intuitive understanding of the meaning of the principle can be provided. Simple calculations lead to an approximate uncertainty relation $\Delta x \cdot \Delta p \approx \hbar / 2$.

![Figure 3. GeoGebra simulation for the single slit diffraction](image)

We pay particular attention to making clear that the uncertainty principle should be interpreted as an intrinsic limitation to the information about complementary variables which quantum states can contain, and not as a perturbation of the system due to measurement.

Single photon Mach Zehnder: refuting the classical trajectory concept

The Mach-Zehnder interferometer can be used to provide a convincing proof of the untenability of the classical trajectory concept. In its base form, the experiment is used with the two arms having the same optical path length, so no phase shift is introduced between the two photon paths. In this case, one of the detectors (Detector 2 in figure 3) has zero probability of detecting the photon, while the other detects the photon with certainty.

The main point of interest in the experiment is to compare this result to what happens when either one of the two arms has been blocked: in this case, detectors A and B have the same probability of detecting the photon, since no interference happens. Thus one can conclude that results are incompatible with the hypothesis that, in the experiment with the full setup, the photon has gone through only one of the two possible paths. Once the idea has been introduced, we make a comparison to the case of two slit interference: indeed, in that case also the interference pattern is statistically incompatible with the two separate diffraction figures which would be obtained if only one of the slits was open. In this way students can reinforce the idea...
that the quantum object going through all possible paths simultaneously is a general feature of the theory, and not connected to the Mach-Zehnder setup only; however, we found that first reducing the outcome possibilities to only two detectors, rather than a continuous screen, has educational advantages for many students.

In our simulation (Figure 4) of the Mach-Zehnder interferometer, the Feynman approach allows to obtain an intuitive understanding of the experiment, in its possible setups.

![Figure 4](image)

**Figure 4.** GeoGebra simulation for the Mach-Zehnder interferometer, with the sum of phasors for the relevant photon paths represented in the right window. Checkboxes allow to compare different situations, highlighting the impossibility of assigning a definite path to the photon.

The optical path of one of the arms can also be varied in the simulation through the insertion of a dielectric film (reflection at the interface is neglected in this case) of variable width. This offers the possibility for students to actually compute probabilities of detection, in a situation which is slightly different from the usual case of two slit interference.

**Zhou-Wang-Mandel Experiment: in depth analysis of the problem of measurement**

The Zhou-Wang-Mandel (ZWM) apparatus (Zhou et al., 1991) (figure 5) is a two way, single photon interference setup where “which way” information is collected in a non-destructive manner through a clever use of nonlinear crystals. The main result of the ZWM experiment consists in proving that the modification in the final outcome of an experiment due to an intermediate measurement, which is peculiar to quantum physics, should not be thought in terms of a disturbance, but of information acquired or recorded about the system. The language of the Feynman approach is particularly appropriate for summing up the lessons that can be drawn from the experiment. In particular, when the expression “all possible paths” is interpreted to mean “all paths compatible with the information about the system”, the idea of “wave function collapse” caused by a measurement, even if of non-destructive nature, is automatically retrieved.
Another purpose of discussing the ZWM experiment is to introduce a generalization of the concept of “path”. In fact, what is to be summed in the sum over paths approach are not necessarily only the possible word-lines of a single particle, but more generally, all possible undistinguishable processes leading to the same experimental outcome: undistinguishable, in the sense that no information can be retrieved about which one of the processes has happened. In fact, the possible paths leading to interference in the ZWM setup are not paths of the “same” photon, but of photons emitted by two different nonlinear crystals, which can be made undistinguishable. This point is even more evident in the famous Hong-Ou-Mandel (Hong et al., 1987) apparatus, where two different, but undistinguishable photons, are sent in the two inputs of a beam splitter. The possible outcomes of the experiment which only differ by an exchange of the two quantum objects interfere destructively, leading (because of a property of beam splitters to produce a $\pi$ phase loss for reflection at only one of their inputs) to the counterintuitive result of having no coincident clicks in the two detectors. An immediate consequence of this generalization is the possibility of introducing the schematic representation of processes in terms of Feynman diagrams.

**The limit of geometrical optics**

When the wavelength of the quantum object becomes much smaller than the relevant length scales, the sum over paths approach reproduces the results of geometrical optics. In fact, the dominant path for the photon in this limit is the one predicted by Fermat’s principle. Simulations in which the wavelength of the quantum object can be interactively varied are essential at this stage; examples include light refraction at an interface and parabolic mirror reflection. The first one is represented in figure 6: an isotropic light source is placed near an interface between two media, with a detector placed beyond the interface. The shape of the “Cornu spiral” which appears in the right graphic window shows that the paths nearby to the one of minimum time give a larger contribution to the final amplitude. Also, the minimum time path becomes more and more dominant, as the wavelength decreases with respect to the source-detector distance. Paths which are very far from the ray of geometrical optics give essentially no contribution, since they go round in “curls” at the ends of the spiral.
Figure 6. GeoGebra simulation for the refraction of light at an interface. In the right window the phasor arrows can be seen to form the characteristic “Cornu spiral”.

Test with student teachers

We carried out a preliminary study with a group of 12 student teachers (ST). Most of the members were non-physicists (mathematicians or engineers), and their background in physics varied. 3 out of 12 ST had never previously had any formal training in modern physics; 5 had followed only one college course, 4 more than one course. We proposed our sequence in a course which lasted 8 hours, divided in session of two hours each, and included presentation of the material, student exploration of simulations with guided activities, open discussion about conceptual themes, and home exercises to be solved through an online discussion. In the first and final sessions, pre- and post-tests were proposed.

Pre-test data

Data from the pre-test showed ST to be completely unfamiliar with a description of light in terms of photons, and very confused about conceptual aspects of quantum theory. 11 ST out of 12 were unable to provide a description of the two slit experiment in terms of photons. Most ST could not name any differences between classical and quantum physics, and none of them was able to provide an entirely satisfying definition of the uncertainty principle. 10 ST provided an inadequate picture of wave particle duality.

Data from post-test

The main aspects we probed in the post test were:

- Whether students were comfortable with quantum model of light, and could explain interference phenomena in terms of individual photons.
- Whether students had reached a satisfying level of confidence and an appropriate language in discussing conceptual and epistemological themes of quantum physics.
- Whether students could use the model to compute probabilities of detection in simple cases.
In a post-test exercise very similar to a pre-test one, requiring to interpret a slightly modified two slit experiment in terms of individual photons, ST provided accurate descriptions and showed a noticeable confidence with the sum over paths language:

"In terms of paths, all possible paths passing through A experience a phase change of when the film is applied. So, where before the vectors were in phase and the probability of detection was maximum, now the vectors are in phase opposition and give probability P=0."

When asked to directly discuss the role of measurement in quantum physics, comparing it to its role in classical physics. Answers produced by ST were, in general, extremely satisfactory, displaying a precise and secure use of language and concepts. Several students mentioned that, in the Feynman picture, acquisition of information on the system restricts its possible paths to only those compatible with the acquired information, which is a formulation of the “wave function collapse” concept. ST convincingly made the point that the very fact of acquiring information about the system, and not some sort of disturbance, is responsible for the restriction in the possible paths.

"If I have a way of knowing which slit the photon goes through, the interference pattern is lost. And this would happen even if I were a “perfect” observer, perfectly “transparent” to the photon passage."

Finally, open and multiple choice exercises showed that ST had acquired sufficient mastery of the sum over paths approach to be able to compute detection probabilities. Online discussion of assigned exercises revealed that one issue several students were confused about was the normalization of probabilities. Thorough discussion of the Mach-Zehnder interferometer, which only has two possible outcomes, and additional exercises exploring this particular problem were useful strategies for dissipating their doubts.

Conclusions

We propose an educational path meant to provide students with an unifying picture of light based on its quantum model, and to introduce them to the most important conceptual and epistemological themes in quantum theory. Although not discussed in this paper, the path has been straightforwardly extended to a comprehensive introduction to quantum theory with the introduction of massive particles, and the discussion of bound systems and quantization from the point of view of Feynman paths (Onorato, 2011; Malgieri et al., 2014).

Feynman’s sum over paths approach allows us to offer students a visualization of the quantum model of light which is not misleading and does not contain hybrid quantum-classical elements. We use the sum over paths method both as a computational tool and as a conceptual point of view, from which we analyze some epistemological themes of quantum physics, aiming at making them clearer and more acceptable for students. We discuss modern experiments in quantum optics carrying a deep conceptual meaning, and allowing to introduce a modern, information based foundational point of view.

Based on the results of this study, our approach appears to be very promising, leading ST to take hold of quantum concepts and acquire an expert-like language in a very short time. Students appear satisfied by the internal models they create using Feynman’s approach.

Our proposal makes extensive use of interactive simulations, which we designed using the open source software GeoGebra. Simulations are built on an interface which many teachers already
use in their educational practice. This can encourage them to modify the provided examples, adapting them to their own needs.

References


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