An Evolutionary Model of Intervention and Peace☆

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Abstract

Intervention often does not lead to peace, but rather to prolonged conflict. Indeed, we document that it is an important source of prolonged conflicts. We introduce a theoretical model of the balance of power to explain why this should be the case and to analyze how peace can be achieved: either a cold peace between hostile neighbors or the peace of the strong dominating the weak. Non-intervention generally leads to peace after defeat of the weak. Cold peace can be achieved with sufficiently strong outside intervention. The latter is thus optimal if the goal of policy is to prevent the strong from dominating the weak.
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1. Introduction

One of the facts of warfare is that victory in battle weakens the opposition making further victories easier. There are exceptions in conflicts over large geographical areas - for example the French and German invasions of Russia failed despite initial successes due to the overstretch of supply - but in regional conflicts over good terrain, absent outside intervention typically one side eventually achieves enough success that it is ultimately able to win the war. Outright victory tends to lead to peace, albeit the peace of the strong ruling the weak. Examples of this are the Union victory in the U.S. Civil War, the defeat of Napoleon and the defeat of Germany and Japan in the Second World War.

When there is outside intervention the weak may be propped up for a long period of time or even indefinitely leading to prolonged and often very bloody conflicts that may last decades or even generations. This depends in part on the goals of the outside powers and their strength. In many cases they support a balance of power, either for selfish reasons, to assure weak opposition, as in the case of Britain supporting a balance of power in continental Europe over many centuries, or because different outside powers take different sides in a conflict. Two obvious examples of conflicts prolonged over decades by outside intervention maintaining a balance of power are the Vietnam War, and the conflict between Israel and the neighboring Arab countries. By contrast conflicts without outside intervention - such as World War II or the US Civil war - are typically short, lasting on the order of five years before one side wins.

Very strong outside intervention in the form of outside rule can lead to peace, but often preserves the underlying source of conflict so that when outsiders leave war again breaks out. For example, Ottoman rule led to centuries of peace in the Balkans, but with the collapse of the Empire conflicts that had been dormant for generations again broke out. Hindus and Muslims lived in peace under British rule in India, but war broke out the moment the British withdrew. We do not address the issue of the relative desirability of short term peace versus long-term conflict, but instead try to develop a useful model of the length and nature of conflict and how it depends upon outside intervention.

The model we develop is a stochastic model of regional conflicts. Under modest assumptions, absent outside intervention one side will win - and relatively quickly - leading to a hegemony and a long peace - that of the conqueror over the vanquished. By contrast, outside intervention typically supports the weaker side and can lead to a balance of power rather than a hegemony. By doing so it typically prolongs conflict. It does so, however, protect the weak from the strong. Hence there is a trade-off: peace being desirable on the one-hand and the protection of the weak being desirable on the other. We find that when the latter is a priority the level of intervention is a relevant determinant of the nature and length of the conflict, with stronger intervention being generally preferable towards the goal of minimizing casualties.

We first develop a stochastic model of regional conflict, then we discuss the implications in a number of different conflicts historical and contemporary.

2. The Model

We summarize the main ingredients of the model before getting to the assumptions. [do we want to start with this disclaimer (which we have given in the introduction already), or shall we more simply just start with "we study a model of conflict within a particular region where there is a finite number..."]We study a particular region in which owing to absence of significant geographical or other barriers there is no natural protection against enemies. For example, the model is not intended to apply to a mountainous region such as the Balkans where successful invasion and conquest is difficult. Within the region there are a finite number J of potential societies contending the region's land with one another through time. These societies may be nation-states or non-governmental organizations. In any period society j has an integer amount $L_j \geq 0$ of land, and in the region there are $L = \sum_j L_j \geq 3$ units of land in total. At any moment of time not all of these societies may be active in the sense that it may be $L_j = 0$ for some j. The vector of land holdings by different societies $z = (L_1, L_2, \ldots, L_J) \in Z$ (a finite set) is the state of the discrete-time Markovian chain we will study. Evolution of the state z is determined by conflict between societies, and the time unit is such that at most one unit of land changes hands each period. Unfolding of conflict depends, besides current land holdings z, on two characteristics intrinsic to the societies. The first is what we call its *unit power*, a real parameter $\gamma_j > 0$ meant to capture the efficiency of a society in conflict. Unit power and land holding together determine the *aggregate power* $\varphi_j = \varphi(\gamma_j, L_j)$ of society j representing the overall ability of a society to prevail in conflict. The second attribute of a society is the *stability* of its institutions, modeled as a binary parameter $b_j \in \{0, 1\}$ with $b_j = 1$ indicating stability and $b_j = 0$ being labeled as unstable. A stable society is made up of individuals who are subject to rules and social norms, understand well the environment they are in and satisfy incentive constraints. It can be thought of as an equilibrium of an underlying social game. By contrast unstable societies represent societies in disequilibrium in which individuals might not satisfy incentive constraints, may not agree on rules, and may be subject to change as individuals learn more about their environment. Such a society can be turbulent and rampageous.

The last element of the model is the presence of exogenously given *outside forces,* with *intervention power* φ_0. These outsiders are assumed to be protected by geography, climate or sheer strength from action by the region in question. For example in the 17th-20th century Great Britain was an outside force with respect to continental Europe being well protected by the natural barrier of the English channel and the strength of British sea power. Currently the U.S. and Russia are outside forces with respect to the Middle East, being protected by distance, the ocean (in the case of the U.S.) and by military strength from Middle Eastern societies.[3] Outside forces have a state-dependent *intervention policy* given by $\pi(j, z)$, the probability that society j is *reinforced* by the outside forces in state z. These outside forces can reinforce only one society at a time. If society j is reinforced its *combined power* is $\phi_j = \varphi_j + \varphi_0$, otherwise it is $\phi_j = \varphi_j$. The corresponding vectors are $\varphi = (\varphi_j)_{j=1}^J, \phi = (\phi_j)_{j=1}^J$. We now provide the details of the model.

[3] Note that we are not considering here terrorism - which on the scale of conflict it is relatively minor.

2.1. Types of Societies and Intervention

Groups of individuals highly committed to a cause - revolutionaries or fanatics - generate a great deal of unit power on account of their willingness to forgo the amenities of ordinary life. On the other hand such groups tend to be unstable as individual willingness to forgo amenities seems to be a short-term affair - the image of the successful elderly revolutionary living in splendor is a familiar one. The implication is that the strongest societies as measured by unit strength are not the most stable. Formally we make

Assumption 1. *(a) there are both stable and unstable societies; (b) the greatest unit power γ_j is unstable:* $\overline{\gamma} = \max_{j|b_j=0} \gamma_j > \max_{j|b_j=1} \gamma_j = \underline{\gamma}$

We next turn to aggregate power $\varphi_j = \varphi(\gamma_j, L_j)$, recognizing that in warfare bigger is better. On a per capita basis an independent Hong Kong would no doubt generate far greater military strength than China, but if it came to war the outcome could hardly be in doubt. The natural assumption is that aggregate power is increasing in both unit power and size as measured by land holding. In addition societies that are *inactive* - that hold no land - are mere templates for societies that might exist but do not currently - hence they cannot generate any aggregate power at all. Formally

Assumption 2 (Aggregate Power). $\varphi(\gamma_j, 0) = 0$, *$\varphi(\gamma_j, L_j)$ is strictly increasing in L_j and for $L_j > 0$ strictly increasing in γ_j; moreover in the latter case as $\gamma_j \to \infty$ we have $\varphi(\gamma_j, L_j) \to \infty$.*

It may be useful here to think of the simplest and prototypical functional form, the multiplicative one: $\varphi(\gamma_j, L_j) = \gamma_j L_j$.

Conceptually *active societies* that have positive land holdings are distinct from inactive societies. Active societies may become inactive due to military defeat represented by the loss of all land. On the other hand an inactive society may become active when an active society loses land: for example, the loss of land may be due to the emergence of a new society with different institutions on that land.

The allocation of land determines the amount of competition between active societies. It is useful to distinguish three levels of competition. We say that z is a *hegemony* of j if society j is the only active society, that is $L_j = L$. We say that z is *binary* between j, k if these are the only two active societies - going head-on-head - so that $L_j, L_k > 0$ and $L_j + L_k = L$. Otherwise when there are three or more active societies we say that z is a *balkanization*.

Intervention Policy

We next describe the intervention policy of the outsider or outsiders. We treat these relatively abstractly and consider three different issues: intervention on behalf of inactive societies, intervention against hegemonies, and "balance of power" interventions. Note that we are agnostic as to whether there is a single outsider or several different outsiders.

With respect to inactive societies it is certainly possible to support revolutionary or dissident groups that as yet are too small to control land. However we assume that the intervenor has little control over which group it supports. Or to put it differently: the intervenor may foment dissension leading to revolt but has little control over the nature of the group that revolts. This can be thought of as informational: it is difficult for outsiders to reliability evaluate the "true" goals of revolutionary groups.

With respect to hegemonies we assume that there is at least some chance that the outsiders will intervene in an effort to break up the hegemon - indeed it seems in practice that this probability is quite high, for hegemons are rarely popular.

With respect to "balance of power" interventions we assume that the outsiders wish to prevent hegemony or at least that they prefer to intervene in favor of the weak over the strong. The strong, after all, have little need of outside assistance. Formally, we make

Assumption 3 (Intervention Policy). *(a) If j, k are inactive then $\pi(j, z) = \pi(k, z)$.*

(b) If z is a hegemony of j then $\pi(k, z) > 0$ for $j \neq k$.

(c) If z is a Balkanization and society k has the greatest aggregate power, that is, φ_k is the unique largest element of φ, then $\pi(k, z) = 0$;

(d) *If z is binary between j, k then there exist thresholds $\overline{L}_{jk}, \overline{L}_{kj}$ with $\overline{L}_{jk} < L - \overline{L}_{kj}$ and such that if $L_j \leq \overline{L}_{jk}$ then $\pi(j, z) = 1$, and if $L_j \geq L - \overline{L}_{kj}$ then $\pi(k, z) = 1$.*

Part (a) captures the idea that the outsiders cannot distinguish between dissident groups. Part (b) is our assumption that there is a chance that an effort will be made to undermine hegemons. Parts (c) and (d) are the "balance of power" intervention assumption and take different forms in the case of Balkanizations and binary states. In the case of a Balkanization part (c) says that at the very least the outsider should not favor the strongest society. The binary case is more structured: part (d) says that the outsiders are assumed to follow a threshold rule with respect to land holding - intervening on behalf of a society whenever their land holding falls below a certain threshold. This is consistent with different outside forces intervening on different sides provided they do not do so simultaneously. We allow the threshold to be 0 meaning that there is no intervention at all on either or both sides. Understanding the role of intervention policies as represented by $\overline{L}_{jk}, \overline{L}_{kj}$ in determining outcomes is one of our primary goals.

Notice that in the j, k binary case we may write combined power of j as $\phi_j(L_j)$ since unit power γ_j is determined by j while intervention is determined by L_j (because $L_k = L - L_j$ and the others are zero).

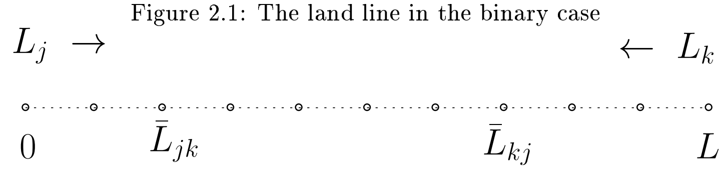
For binary states between j, k it is convenient to take the state variable as the scalar $L_j(z)$ with land for the other society implicitly determined as $L_k(z) = L - L_j(z)$. The situation can be visualized as a line with points from zero to L where L_j is measured from left to right and L_k is measured from right to left. See figure 2.1. The inequality $\overline{L}_{jk} + \overline{L}_{kj} < L$ means that $\overline{L}_{kj}$ appears on the right of $\overline{L}_{jk}$.

Remark.

2.2. The Markov Process

We model a dynamic process of conflict between societies with a family of Markov processes on the state space Z with transition matrices $P_\epsilon(z_t | z_{t-1})$, indexed by $\epsilon > 0$. The ϵ-process is a

Figure 2.1: The land line in the binary case



In the picture at $\overline{L}_{jk}$ it is $L_j = 2$, $\overline{L}_{kj}$ is with $L_k = 3$.

perturbation of a given process P_0 in the sense that for all z one has $\lim_{\epsilon \to 0} P_\epsilon(z_t|z_{t-1}) = P_0(z_t|z_{t-1})$. The parameter ϵ may be interpreted as the chance of success when facing overwhelming opposition - for example, the chance that a small band of rebels succeeds in overthrowing a large and powerful monolithic state - and we shall be interested in the case where ϵ is small but positive. Two anecdotes may help visualize. On December 2, 1913 in Alsace-Lorraine a shoemaker Karl Blank laughed at German soldiers, and was beaten and paralyzed. Subsequently there were protests of up to 3,000 people: we may think of ϵ as the probability that this "rebellion" would succeed in wresting control of the Alsace-Lorraine region from the control of the German Empire - needless to say it did not. However, unit power of the incumbent γ_j is an important element as well: it was rather large in the case of the German Empire. In contrast on June 14, 1846 thirty three people successfully took control of the Mexican State of California: in the case of Mexico γ_j was very small.

A fundamental mathematical tool in the study of such a parametrized system is the notion of *resistance* (see, for example, Young (1993)), which measures how small is small. Specifically, a transition with probability P_ϵ is said to have *resistance* $0 \leq r \leq \infty$ if there are constants $0 < C < 1 < D < \infty$ such that $C\epsilon^r \leq P_\epsilon \leq D\epsilon^r$. Higher resistance (to the state changing) roughly means much lower probability of the transition. In particular zero resistance is equivalent to positive probability with respect to P_0 and infinite resistance means zero probability for all values of ϵ. The probability P_ϵ of a transition with positive finite resistance r goes to zero like ϵ^r as $\epsilon \to 0$. In specifying the model P_ϵ it is convenient to work directly with resistances rather than probabilities.

The Markov process is induced by the intervention policy described above and two resistance functions. The first describes the probability that an active society j loses a unit of land, through a *conflict resolution function* indicating the resistance $r_j(z) < \infty$ to j losing one unit of land - that is the resistance of a transition from z to a state where j has one less unit of land. It is also useful to define $r_0(z)$ as the resistance to no society losing land - in effect the inverse of the probability that nothing happens. As it seems there should be an appreciable chance that nothing happens we assume that $r_0(z) = 0$. The second resistance function describes the probability that if j loses land the land goes to society k, through a *land gain resistance function* $\lambda_{kj}(z)$. This should be read as the resistance to k getting a unit of land from j conditional on j having lost a unit of land.

2.3. The Conflict Resolution Function

The conflict resolution function, that is, resistance of a society to losing land, should depend on its stability and on the military strength of the particular society and its rivals.

With respect to stability we argue that unstable societies have no resistance to losing land. We view an unstable society as one with a non-negligible degree of randomness in their members' behavior - charismatic leaders may arise, populist nonsense may be believed and so forth. These events then lead to a chance that the unstable society will collapse - at least in the sense of losing a unit of land either to some new social arrangement or to being absorbed by some other active society.

It must be emphasized that the assumption that unstable societies have no resistance to losing land in combination with the existence of stable societies has important and not entirely obvious consequences. By allowing unstable societies we avoid assuming that each society is always in equilibrium. It is, however, the case that equilibrium in the sense of a state in which active societies are all stable is reached rapidly and that only these states can persist for any length of time. That is: most of the time we see only stable societies. This a sort of "global convergence to Nash equilibrium" result and indeed with a more elaborate model, such as that in Levine and Modica (2013), we can model the play and incentives of individual players and formally consider a decentralized learning procedure in an environment of conflict. The discovery that this type of stochastic process converges globally to equilibrium (in the sense of being in equilibrium most of the time) is due to Foster and Young (2003). It is known from the work of Hart and Mas-Colell (2003) that the only decentralized learning procedures that have this global convergence property are stochastic, and they give a deeper discussion of the types of stochastic learning processes that do have this property in Hart and Mas-Colell (2013).

From an economic perspective we think of rapid convergence to stability as a good description of reality: we observe that even in highly unexpected and disrupted situations - such as refugee camps - people seem to quickly find modes of behavior that are sensible for the new environment.

Other than descriptive realism it might seem that little is gained by replacing the assumption of "always in equilibrium" with an assumption that implies "quickly in equilibrium." However, as we shall see, while unstable societies are rarely seen they never-the-less play a key role.

With the assumption that unstable societies have no resistance to losing land we may write the resistance of society j to land loss as

$$r_j(z) = b_j \cdot r^0(\phi_j, \phi_{-j})$$

where r^0 is called the *basic* resistance and states that the resistance of a stable society to losing land depends on its combined power and that of its rivals. We now wish to consider what the basic resistance should look like. Two obvious assumptions are anonymity - that it is the strengths not names of societies that matters - and monotonicity - that greater strength and weaker opponents means greater resistance to losing land. We also want to capture the idea that we are modelling a region within which geographical and other barriers are weak. We do so by assuming the weakest society - unprotected by barriers as it is - has no resistance to losing land. We do not rule out the possibility that more powerful societies may also have no resistance to losing land. When geographical barriers are strong it may be that neither of two nearly equal powers has a realistic chance of taking land from the other. It is no mystery why Switzerland is Switzerland or the Balkans are Balkanized - this is a matter of mountains and rugged terrain ill-suited for invasion. Our goal here is to study conflict that takes place in a region within which there are not important geographical barriers.[4] Formally

Assumption 4 (Basic Resistance). *(a)* (anonymity)$r^0(\phi_j, \phi_{-j})$ *is independent of the order of the societies in* ϕ_{-j}*;*

(b) (monotonicity) $r^0(\phi_j, \phi_{-j})$ *is weakly decreasing in* ϕ_{-j} *and weakly increasing in* ϕ_j*; whenever it is positive it is strictly decreasing in* ϕ_{-j} *and strictly increasing in* ϕ_j*, finally, as* $\phi_j \to \infty$ *we have* $r^0(\phi_j, 0) \to \infty$*;*

(c) (weak barriers) *for some* $\lambda > 0$ *if* $\phi_j > 0$ *is such that* $\phi_j \leq (1+\lambda)\phi_k$ *for all* k *with* $\phi_k > 0$ *then* $r^0(\phi_j, \phi_{-j}) = 0$.

Parts (a) and (b) are clear; part (c) states that the weakest active societies have no resistance to losing land. An example of a function satisfying these properties is

$$r^0(\phi_j, \phi_{-j}) = \max\{0, \frac{\phi_j}{1 + \sum_k \phi_k} - (1+\lambda) \min_\ell \frac{\phi_\ell}{1 + \sum_k \phi_k}\}$$

Our final assumption concerning conflict involves the strength of unstable societies. We refer to the strongest unstable societies a *superzealots* bent as they are on conquest and victory at all costs - think here of Ghengis Khan, Tamerlane or Lenin. Such societies may burn out quickly but if they survive they do have an appreciable chance of conquering a unit of land. To state the formal assumption recall that $\overline{\gamma} = \max_{j|b_j=0}$ is the greatest strength of an unstable society - that is the strength of superzealots - and that $\max_{j|b_j=1} \gamma_j = \underline{\gamma}$ is the greatest strength of a stable society. We already assumed $\overline{\gamma} > \underline{\gamma}$. We now strengthen this to assert that when superzealots are pitted against the strongest stable society - aided by outside forces even - there is never-the-less zero resistance to that stable society to losing land. Formally

Assumption 5. $r^0(\varphi_j(\underline{\gamma}, L-1) + \varphi_0, \varphi_k(\overline{\gamma}, 1), 0) = 0$

An important case of resistance is *hegemonic resistance*, that is the resistance of a hegemony holding all the land to a spontaneous disruption - invasion by outsiders or rebellion by insiders. This necessarily depends only on the strength of the hegemon and the strength of the outsiders intervening on behalf of inactive societies - that is in fomenting dissent.

Definition 1 (Hegemonic resistance). $r_j^h \equiv b_j \cdot r^0(\varphi(\gamma_j, L), \varphi_0, 0)$

In a similar vein, in the case of binary states between j, k it is convenient to abbreviate the basic resistance $r^0(\phi_j, \phi_k)$ as depending only on the strengths of the two combatants, all the other strengths being zero.

[4] It is also the case that particular military technology may favor the defense so that even in the absence of geographical barriers invasion is impractical. However historical examples of this type are difficult to find. One example may be the stand-off between the Roman Empire and Persia: the two powers used incompatible military technologies, neither able to defeat the other.

2.4. Land Gain Resistance

When land is lost, to which society does it go? First, as a purely technical matter is should not go back to the original owner since otherwise it it not lost, so we assume $\lambda_{kk}(z) = \infty$, and that otherwise it is finite. If land is lost in conflict and there is another stable society it seems likely that the occupants of the "lost" land will wind up being absorbed by some other society that is by some measure "successful." By the same token we assume that it is very unlikely that a new society will spring up on the "lost" land. These are our basic assumptions. However there are two other considerations. First a hegemony - by definition - can lose land only to an inactive society. In keeping with the basic idea that newly emerged societies have an element of unpredictability we assume that all inactive societies have an appreciable chance of entering. Second if the society that loses land is unstable this is as likely to be due to experimentation with something different as a loss due to conflict with or conquest by another society - and again remaining agnostic about the nature of the experiment we assume that all societies have an appreciable chance of arising on the land lost from an unstable society. Formally

Assumption 6. *Let $\overline{r} = \max_{j,z} r_j(z)$ be the largest possible resistance to losing land.*

(a) if $b_j = 1$ and $L_j(z) > 0$ so j is stable and and active then $\lambda_{jk}(z) = 0$

(b) if z is binary with two stable societies j, k then for an inactive society ℓ we assume $\lambda_{\ell j}(z), \lambda_{\ell k}(z) > L\overline{r}$

(c) if k is a hegemony then $\lambda_{jk}(z) = 0$ for all $j \neq k$

(d) if k is unstable then $\lambda_{jk}(z) = 0$ for all $j \neq k$

Part (a) is the assertion that active stable societies (if any) have an appreciable chance of getting land. Part (b) says that the chances that inactive societies get the land in a binary configuration with two stable societies is very small (that is, the resistance is very high). Part (c) says if a hegemony loses land it is uncertain which inactive society will arise as a result. Part (d) is the similar assertion for land lost by unstable societies.

3. Theoretical Results

Using results from the theory of Markov chains we will characterize which configurations will "typically" occur over long periods of time. These are of two types: hegemonies and balance of power[did we say we call binary "balance of power"?]. Our focus is on characterizing how frequently these different configurations will be observed and particularly on the types and circumstances of balance of power. We shall see that there are two types of stable balance of power configurations. One is a *cold peace,* where a single unit of land changes hands back and forth. It may be thought of as corresponding to border skirmishes - for example, the recent conflict between Israel and Lebanon which occasionally flares into the firing of rockets over the border or small border incursions. Such a continuing conflict is relatively low key and "peaceful." By contrast in a *prolonged war* the two sides fight back and forth losing and gaining substantial amounts of land. The current civil war in Syria might be one such example.

3.1. Description of Recurrent States

Markov chains have two types of states - transient states which with probability one after some length of time are never seen, and recurrent states which with probability one recur infinitely often. We will see that P_ϵ is ergodic so that all states are recurrent, but only the recurrent states of the process P_0 corresponding to $\epsilon = 0$ are frequently seen when ϵ is small but positive. By *transient* and *recurrent states* we always mean transience or recurrence with respect to P_0. As they are important in P_ϵ as well as P_0 it is a useful first step to consider where the recurrent states lie. There are three possibilities: hegemonic states, single balance of power segments and paired balance of power segments. Each recurrent state z has associated with it two numbers: $R_z > 0$, called the radius of the state which represents roughly speaking the resistance to escaping "far" from that state; and $M_z \geq R_z$, the modified radius which is a broader measure but the same idea. The dynamics (for $\epsilon > 0$) can be well described by these numbers as we shall subsequently explain - in particular bigger values of these radii means that the corresponding states are seen more frequently. Before doing the dynamic analysis we first describe the three types of recurrent states and in each case explain how to compute R_z, M_z; in the case of transient states it will always be the case that $M_z = R_z = 0$.

3.2. Transient States and Hegemony

Binary states with at least one unstable society and all balkanizations are transient. This is proved in Lemma 8.

For a hegemonic state z where the hegemon is society j the two radii are the same and equal to the hegemonic resistance: $M_z = R_z = r_j^h$; these states are will be either recurrent or transient as the radius is positive or zero, but Assumption 4(b) implies the radius is positive for high enough φ_j.

3.3. Balance of Power Segments

Most interesting from our point of view are the balance of power segments, which may be either single or paired.[5] They occur always between two active stable societies j, k. There are two types of segments. *Short* segments consist of two states, either with $L_j = \overline{L}_{jk}, \overline{L}_{jk} + 1$ or $L_j = L - \overline{L}_{kj} - 1, L - \overline{L}_{kj}$. *Long* segments are all the states between $L_j = \overline{L}_{jk}$ and $L_j = L - \overline{L}_{kj}$ inclusive. Note that a segment can be both short and long if $\overline{L}_{jk} + \overline{L}_{kj} = L - 1$. A paired segment is formed by the two short segments. Depending on the level of intervention some of these segments are recurrent and have positive radius and some are transient and have zero radius. We distinguish four levels of intervention on behalf of j against k in the following

Definition 2 (Intervention strength)**.** 1. *Very strong.* Intervention takes place when resistance is positive in the absence intervention: $r_j^0(\varphi_j(\overline{L}_{jk}), \varphi_k(L - \overline{L}_{jk})) > 0$

2. *Very weak.* Intervention is insufficient to give positive resistance: $r_j^0(\varphi_j(\overline{L}_{jk}) + \varphi_0, \varphi_k(L - \overline{L}_{jk})) = 0$

[5] We call them segments because they appear so in the graphical representations we use, see Figure 2.1.

For the remaining cases we assume that #1and #2 do not hold, that is $r_j^0(\varphi_j(\overline{L}_{jk}), \varphi_k(L - \overline{L}_{jk})) = 0$ and $r_j^0(\phi_j(\overline{L}_{jk}), \phi_k(L - \overline{L}_{jk})) > 0$:

3. *Strong.* When j gains a unit of land above the threshold (thus losing support) the opponent has zero resistance to losing land: $r_k^0(\phi_k(L - \overline{L}_{jk} - 1), \phi_j(\overline{L}_{jk} + 1)) = 0$

4. *Weak.* When j gains a unit of land above the threshold the opponent has positive resistance to losing land: $r_k^0(\phi_k(L - \overline{L}_{jk} - 1), \varphi_j(\overline{L}_{jk} + 1)) > 0$

A segment is recurrent - proven in Appendix 2 - if the resistance of moving between states in the segment is zero and of leaving the segment is positive. For the moment we will call segments of this type recurrent with the understanding that the name is not justified until Theorem 2. In Appendix 1 we characterize the relationship between intervention and the existence of recurrent segments of different types and report the results here:

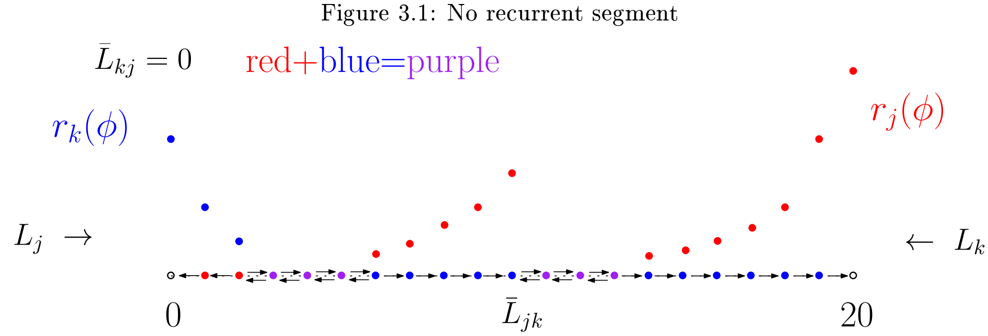
Theorem 1. *Existence, if any, of recurrent segments depending on the type of intervention on behalf of societies j and k can be summarized in the following table (where land is expressed in units of L_j):*

	strong k	*weak k*	*very weak/none/very strong k*
strong j	*long*	*short $L - \overline{L}_{kj}$*	*none*
weak j	*short $\overline{L}_{jk}$*	*paired short*	*short $\overline{L}_{jk}$*
very weak/none/very strong j	*none*	*short $L - \overline{L}_{kj}$*	*none*

No Balance of Power Segment

In case there is no balance of power segment all the states z that are binary between j, k are transient and the radii are zero: $M_z = R_z = 0$. This case is illustrated in figure 3.1. As always L_j is measured from left to right and L_k from right to left; arrows denote zero-resistance transitions.

Figure 3.1: No recurrent segment

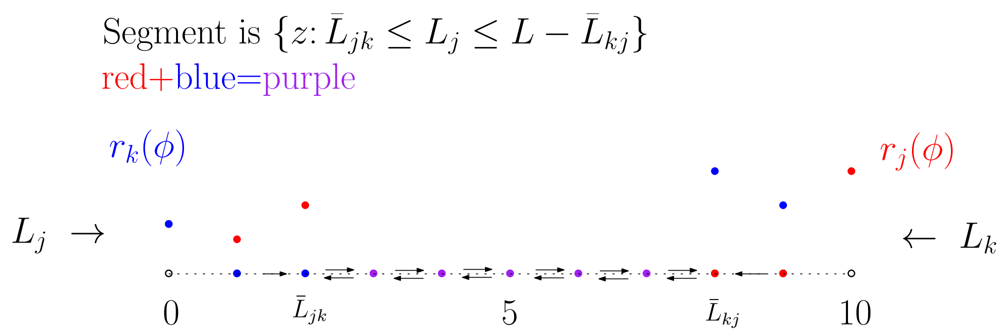


Single Balance of Power Segment

We now consider how to compute the radius and modified radius of single recurrent segments between j, k. In this case we can always compute a *left radius* R_{jk}^{ℓ} as follows. Start at the left endpoint $\overline{L}_{jk}$. Reduce the land holding of j one unit at a time, that is from $\overline{L}_{jk}$ to $\overline{L}_{jk}-1$ and so on. Each time compute the resistance to the land loss - we know from Theorem 1 that since this is a recurrent segment that the first step at least has positive resistance. Add these numbers together and continue until j has lost all their land and become inactive. That is, we take $r_j^0(\phi_j(\overline{L}_{jk}), \phi_k(L-\overline{L}_{jk}))$, add to it $r_j^0(\phi_j(\overline{L}_{jk}-1), \phi_k(L-\overline{L}_{jk}+1))$ and continue until all land is lost. This gives the left radius. Similarly we compute the *right radius* R^r_{jk} by adding up resistances for k starting at the right endpoint where $L_k = \overline{L}_{kj}$ and reducing the land holding of k one unit at a time until hegemony is reached. Again since the segment is recurrent at least the first step has positive resistance. The radius is the smaller of the left and right radius: for z such that $\overline{L}_{jk} \le L_j(z) \le \overline{L}_{jk}+1$ we have $M_z = R_z = \min\{R^\ell_{jk}, R^r_{jk}\}$ for all z in the segment.

The case of a single recurrent long segment is illustrated in figure 3.2 below.

Figure 3.2: Single long balance of power segment



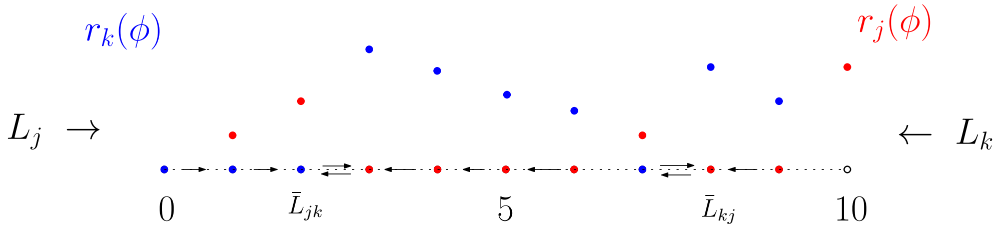
[Comment, I'm not sure where it goes if anywhere because we don't really talk about this. If there are two intervenors with conflicting interests, I am thinking of Russia and the US, the theorem says that against a strong k (Russia) if you play weak j you reach a short segment while if you play strong you produce a long segment. This may be worse, but not always. Because if the long segment in the strong/strong case is also short it is central, while a weak/strong short segment is on one side. And the central one is likely less bloody, because the parties have less hope to prevail hence are less willing to invest in warfare, while in the segment on one side the stronger has victory in sight and pushes more fiercely. So the result does not imply that you should play weak against a strong opponent intervenor.]

Paired Balance of Power Segment

There are paired balance of power segments when both short segments are recurrent. This is illustrated in figure 3.3 below.

The calculation of the radius is similar to that of a single short segment, except that now the right radius of the left segment is computed stopping when the other segment is reached rather than continuing all the way to hegemony. That is, we compute the *right paired radius* R^{rr}_{jk} by adding up resistances for k starting at the right endpoint, where $L_j = \overline{L}_{jk}+1$ and reducing the land holding of k one unit at a time until we reach the left endpoint of the other segment, where $L_k = \overline{L}_{kj}+1$. Again at least the first step has positive resistance. The radius of the left segment is

Figure 3.3: Paired balance of power segment



now the minimum of the left radius and right paired radius: for z such that $\overline{L}_{jk} \leq L_j(z) \leq \overline{L}_{jk} + 1$ we have $R_z = \min\{R^{\ell}_{jk}, R^{rr}_{jk}\}$. Similarly for the right segment we compute the right radius to k's hegemony as before and a *left paired radius* $R^{\ell\ell}_{jk}$ which is the total resistance - going left - from $L_k = \overline{L}_{kj} + 1$ to $L_j = \overline{L}_{jk} + 1$. And for the two states in the segment $R_z = \min\{R^r_{jk}, R^{\ell\ell}_{jk}\}$.

The case of paired segments is different than all other cases in that it is the only case in which the modified radius can be different from the radius. Define $\mathcal{D} \equiv \min\{R^{\ell}_{jk} - R^{rr}_{jk}, R^r_{jk} - R^{\ell\ell}_{jk}\}$. This measures how much more difficult it is to get to hegemony than to the other segment. If $\mathcal{D} \leq 0$ so that it is easier to get to hegemony then for all binary z between j, k we have $M_z = R_z$ just as in the other cases. Finally, if $\mathcal{D} > 0$ so it is easier to reach the other segment than to reach hegemony then for z in the balance of power segments we have $M_z = R_z + \mathcal{D}$. This will measure the relative amount of time the system spends in z, taking account of the fact that both segments are likely to be seen many times before hegemony is reached. As an illustrative example consider a case where $0 < \min\{R^{\ell}_{jk} - R^{rr}_{jk}, R^r_{jk} - R^{\ell\ell}_{jk}\} = R^{\ell}_{jk} - R^{rr}_{jk}$ - so in the left segment $M_z = R^{\ell}_{jk}$, and in the right one $M_z = R^{\ell\ell}_{jk} + R^{\ell}_{jk} - R^{rr}_{jk}$. If $R^{\ell\ell}_{jk} < R^{rr}_{jk}$ then the right segment has smaller M_z than the left one, no matter how large R^r_{jk} is. The right segment may seem the more persistent one if R^r_{jk} is large, because resistance to the right is large and when the system moves to the left segment it goes back with resistance $R^{rr}_{jk} < R^{\ell}_{jk}$. But this does not take account of the fact that small $R^{\ell\ell}_{jk}$ means that the system leaves the segment relatively quickly, while leaving the left segment faces higher resistance - $R^{rr}_{jk} > R^{\ell\ell}_{jk}$ and R^{ℓ}_{jk} is even higher. So the system will spend more time in the left segment. Theorem 2(2) makes this intuition precise.[6]

3.4. Dynamics

We can now describe the dynamics of the system. In Appendix 2 we prove

Theorem 2. *(1) States with $R_z = 0$ are transient, states with $R_z > 0$ are positively recurrent, states with $R_z > 0$ that are hegemonic are absorbing and balance of power segments with $R_z > 0$ are absorbing and starting from any state in the segment all other states in it are hit infinitely often.*

(2) When $\epsilon > 0$ there is a unique ergodic distribution μ_ϵ with a unique limit $\mu_0 = \lim_{\epsilon \to 0} \mu_\epsilon$ and

[6]Further details in Levine and Modica (2016), section 6.3

$\mu_0(z) = 0$ if $R_z = 0$. If $R_z, R_x > 0$ then

$$0 < \lim_{\epsilon \to 0} \frac{\mu_\epsilon(z)}{\mu_\epsilon(x)} \cdot \epsilon^{M_z - M_x} < \infty$$

and there are constants $0 < C < 1 < D < \infty$ such that starting at z [at x? otherwise same z...]the expected hitting time T before reaching a different hegemony or balance of power segment with $R_z > 0$ satisfies

$$C\epsilon^{-R_z} \leq T \leq D\epsilon^{-R_z}.$$

The actual escape when it occurs is short and has expected length bounded above by D.

In words the picture is this. The system spends most of its time at hegemonies or balance of power segments with positive radii. The time it takes to escape from one of these is approximately $C\epsilon^{-R_z}$, but while we will leave and fall back to z many times before escaping, the actual escape when it takes place will be short, no longer than D.[7] If we start at a state with zero radius it only takes about D period to reach a state with a positive radius. The relative time spent at the recurrent hegemonies or balance of power segments is of the order ϵ raised to the difference in the radii, with larger radii being seen for much longer periods of time. Observe from part (2) that only states z with highest M_z have positive probability in the limiting distribution μ_0. These states are called *stochastically stable.*

It might appear that in the balance-of-power case the theory cannot account for brief periods of instability such as Napoleon or Hitler - but this is not correct, because before leaving a recurrent communicating class for another there will be many excursions within the radius.

3.5. Comparative Statics

The stability of z is measured by the radius R_z as laid out in Theorem 2. It is natural then to inquire: what leads to large values of R_z, or put differently, what circumstances favor the stability of a particular state? The conclusion is roughly that internal strength and external weakness favor hegemony, while internal weakness and external strength favor balance of power.

To say this a little less roughly: strong unit power - strong internal institutions - and weak outside forces lead to a high radius for hegemony. By contrast weak unit power of the stronger society, strong unit power of the weaker society and strong outside forces lead to a high radius for balance of power segments. This is

Theorem 3. *(1) If z is a hegemony of j then R_z is increasing in γ_j and decreasing in φ_0.*

(2) If z is in a balance of power segment between j, k, assume to fix ideas that least costly exit from the segment is from the left. That is, for a single segment $M_z = R^\ell_{jk}$ and for a paired segment $M_z = R^\ell_{jk}$ in the left segment and $M_z = R^{\ell\ell}_{jk} + R^\ell_{jk} - R^{rr}_{jk}$ in the right one. Then for sufficiently small perturbations M_z is increasing in γ_j, decreasing in γ_k and increasing in φ_0 .

[7]The leaving and falling back many times is complicated to state precisely and is not given in the Theorem, but follows from the theory developed in Levine and Modica (2016).

(3) There exists a unique $\hat{\varphi}_{jk} > 0$ such that if $\varphi_0 > \hat{\varphi}_{jk}$ then for z in a balance of power segment between j, k we have $M_z > r_j^h, r_k^h$ and conversely if $\varphi_0 < \hat{\varphi}_{jk}$ then for z in a balance of power segment between j, k we have $M_z < r_j^h, r_k^h$.

(4) For given φ_0 if the greatest unit power of a stable society $\underline{\gamma}$ is large enough then only hegemony is stochastically stable.

Proof. Part (1) of the Theorem follows directly from the fact that the radius of a hegemony is the hegemonic resistance $r_j^h = b_j r^0(\varphi(\gamma_j, L), (\varphi_0, 0))$ and Assumptions 2 and 4.

In part (2) we assume the perturbation is sufficiently small that the location of the segment and the target(s) of the least or second least resistance paths do not change. If $M_z = R_{jk}^{\ell}$ the result follows directly from the monotonicity assumption on the basic resistance function.For the case $M_z = R_{jk}^{\ell\ell} + R_{jk}^{\ell} - R_{jk}^{rr}$, the two positive terms move in the correct direction as above; the term R_{jk}^{rr} is on the contrary increasing in γ_k and decreasing in γ_j, but then its negative has the right sign.

Part (3). When the radius is strictly positive it is weakly increasing in φ_0 while hegemonic resistance is decreasing when it is positive. The result follows directly.

Part (4) follows from the fact that Assumption 4 (c) implies that for fixed φ_0 there is a largest γ_j for which balance of power is feasible, hence a greatest radius/modified radius of any balance of power segment independent of how large γ_j is. On the other hand our boundary conditions on monotonicity imply that hegemonic resistance goes to infinity as $\gamma_j \to \infty$ so that the hegemonic resistance - radius - of the strongest stable society must eventually exceed the radius/modified radius of any balance of power segment. □

4. War and Peace in the 20th Century

To focus thinking it is useful to consider a simple case that highlights the main conclusions of the the theoretical analysis. We suppose that j, k are equally strong so that $\gamma_j = \gamma_k$, and that these are large enough that the two societies have positive hegemonic resistance - so that if there is no balance of power the system will reach a hegemony of one of the two and stay there for some time. We assume that the intervention policy is symmetric so that $\overline{L}_{jk} = \overline{L}_{kj}$ and policy is indexed by a single scalar, the land threshold for intervention on behalf of both contenders. We assume that the number of units of land L is odd.[8] Finally we assume that the strength of the intervenor φ_0 is high enough that strong intervention is possible, but that it is ineffective for $\overline{L}_{jk}$ sufficiently small.

Start with $\overline{L}_{jk}$ small. In this case intervention is ineffective - there is no point in intervening when j has become so weak that they have lost anyway. In this case there is no balance of power segment, but rather a hegemony of one society: we refer to this as the *peace of the strong over the weak*. As $\overline{L}_{jk}$ increases, eventually the point is reached where intervention is weak. Now there are two paired short balance of power segments, corresponding to a *cold peace*. As $\overline{L}_{jk}$ increases

[8]It is feasible for the two thresholds $\overline{L}_{jk}, \overline{L}_{kj}$ to be adjacent; this would be ruled out by symmetry if L is even.

further intervention becomes strong and there is now a single long balance of power segment - a *prolonged war*. Hence conflict which was low increases substantially. As $\overline{L}_{jk}$ increases the length of the long segment shrinks reducing the scale of the conflict until eventually $\overline{L}_{jk}$ reaches the center and the long segment is also short and we are again at cold peace. We want to emphasize the non-monotonicity: a weak or sufficiently strong intervention leads to cold peace, but an intermediate intervention leads to prolonged war and it is the costliest in terms of lives and distress of the peoples and economies involved.

The cold peace brought about by sufficiently strong intervention obviously gives rise to the largest radius for any balance-of-power segment. That is, if the goal of the intervenors is to maintain the balance of power as long as possible they should go for a cold peace with strong intervention. This result generalizes beyond the symmetric case: in Appendix 3 it is shown that the balance of power that has the greatest radius is always a short segment with strong intervention - a cold peace. However: intervenors may have goals besides preserving the balance of power - in particular, as seems to be the case with Europe and Russia, they may be concerned about the cost of intervention and hence not intervene in sufficient strength to bring about cold peace.

This describes the long term behavior of the system. In addition transitions between the recurrent states take place - from hegemony to balance of power and back. These also involve conflict and war - but by Theorem 2 these *transitional wars* are relatively short.

4.1. Overview

Table 1 gathers the substantial post World War II conflicts. Cases where one combatant did not occupy any land are excluded as the theory does not apply. For the rest we were unable to come up with a precise algorithm for choosing, but we believe that all significant conflicts are accounted for. In some cases there were several intervention regimes: we discuss those separately below. The data is taken from Wikipedia. The table shows the region, the year in which the conflict began, and the number of years it lasted. Casualties (including civilian casualties) are reported in deaths per 100,000 per year which is the standard unit for reporting, for example, murder rates.[9] To put these numbers in context, note that the overall murder rate for Europe and Asia is about 3, for the entire world about 6, and for Africa about 12 and for the Americas about 16. So, for example, the death rate of 20 in the Sri Lankan civil war (a cold peace) is comparable to the murder rate in the Americas, while the death rate of 380 in the Syrian civil war (a prolonged war) is more than an order of magnitude higher. Following the casualty rates we list the parties and outside intervenors. In cases when war ended due to the withdrawal of intervention we report the "collapse" as the number of subsequent years until one side achieved victory. Finally we record, based on the duration, intensity and nature of intervention, our view of which recurrent class the conflict represents, or if it is transitional in nature. Note that transitional conflicts are consistent with outside intervention

[9]Civilian casualties are the bulk of casualties and there are a wide range of estimates. We used the middle of the range of estimates.

provided that the intervenor is attempting to help one side win and not to preserve a balance of power. Entries in the table are arranged in chronological order within each class.

4.2. Consequences of Intervention

Intervention that either is designed to preserve the balance of power or which does so because of conflicting interests of the intervenors can lead either to a cold peace or a prolonged war. There is a large discontinuity in the amount of harm done in a cold peace and a prolonged war: in a cold peace death rates are on the order of relatively high murder rates, or in some cases lower. Taking Sri Lanka, for example, we see that for 26 years the death rate was about 20, comparable to the highest murder rates in the world. The sum across the 26 years is 520, which is somewhat greater than a transitional war that might last only a year or so but is quite bloody - for example, the breakup of India and Pakistan after the British withdrawal where the corresponding number is 250. Overall a cold peace does not seem to represent much of a savings in terms of casualties over non-intervention and a transitional war - but it does protect the weak.

By contrast a prolonged war has much higher fatality rates - perhaps five times higher than a cold peace and only somewhat less than a transitional war, on the order of 100 or more. But like a cold peace and unlike a transitional war, a prolonged war drags on for decades. From a policy point if we were to take the point of view that, say, Lebanon posed a threat, then keeping it a bloody mess for three decades would surely neutralize that threat - but from a humanitarian point of view it represents a catastrophe. If we are to take a very cynical view of the conflict between Shia and Sunni, especially the current hot war in Syria, as a Western effort to preserve a balance of power that neutralizes the Arab world as a threat - the wave of refugees descending on Europe with the consequent social and political problems shows that such an effort can have unintended consequences.

4.3. Transitional Wars Leading to Hegemony

We observe a number of wars with little outside intervention: these generally result in hegemony and moreover, as the theory predicts, they are relatively short. The only one longer than 1 year is the Iran/Iraq war. Often these transitional wars occur after a change in intervention policy in the form of the withdrawal of outside forces leading to collapse of the balance of power. A number of well known conflicts have this character: after the British withdrawal from Palestine in 1948 war broke out between Israel and the Arab nations: this lasted less than a year. Similarly when Britain withdrew from India conflict broke out between India and Pakistan including conventional warfare over Kashmir. This lasted slightly longer than a year. About a year after Richard Nixon agreed to "peace with honor" in 1973 - meaning actually that he agreed to stop intervening - North Vietnam launched an assault on the South winning the war in about a year. In Eastern Europe after Gorbachev announced that the Soviet Union would cease intervention to the complete dissolution of the Iron Curtain the fall of the Berlin wall came about in a matter of months.

The Rwandan Civil War is an interesting case study in what happens without intervention. The conflict was largely ethnic between Hutu and Tutsi. On April 6, 1994 the plane of the Hutu

Table 1: Significant Post World War II Conflicts

Region	Start	Duration	Casualties	Parties	Intervenors	Collapse	Class
India	1946	1	250	India Pakistan			transitional [1]
Palestine	1948	0.75	700	Israel Arab			transitional
Bangladesh	1971	1	100	Bangladesh Pakistan	India	1	transitional
Iran/Iraq	1980	8	100	Iran Iraq			transitional
Falklands	1982	0.2	0	Argentina UK		0.2	transitional
Rwanda	1994	0.25	6800	Hutu Tutsu		0.25	transitional
Iron Curtain	1945	46	0	Eastern Europe Western Europe	Soviets US	1	cold peace
Sinai	1948	68+	1	Israel Egypt	West Soviets [2]		cold peace
Korea	1950	66	8	North South	China US		cold peace
Sri Lanka	1983	26	20	Government Tigers	India	[3]	cold peace
Nagorno-Karabakh	1988	28	26 [4]	Armenia Azerbaijan	Turkey Russia		cold peace
Vietnam	1955	20	170	North South	Soviets US	1	prolonged war
Sudan	1955	60	330	North Sudan Southern Sudan	Horn of Africa		prolonged war
Angola	1975	27	86	MPLA UNITA	Soviets South Africa	1 [5]	prolonged war
Lebanon	1975	31	400	Shia Christian/Druze	Syria Israel	1	prolonged war
Syria	2011	5+	380	Government Insurgents	Russia, Iran West		prolonged war

Table Notes

1. India did not attempt to preserve a balance of power but helped Bangladesh to win the war.
2. It should be noted that originally the Soviets supported Israel.
3. Indian troops left Sri Lanka in 1990 nineteen years before the Government victory. Prior to sending troops in 1987 the Indian government aided the Tigers through the intelligence agency RAW. It likely that this continued after withdrawal in 1990, but there is no information available about this, so we cannot say when or even if India stopped supporting the Tigers.
4. It is unclear in which population the casualties occurred. Virtually all deaths occurred during the six years of active war beginning in 1988. It is estimated that 28,000–38,000 died in that conflict. The population of Nagorno-Karabakh is only 147,000, but it is highly unlikely the bulk of casualties occurred among that population. We used the average of the population of Azerbaijan and Armenia as our base population.
5. The date at which intervention on behalf of UNITA ceased is unclear. We dated it to May 2001 when DeBeers - the main source of funding and illicit weapons shipments to UNITA - ceased operation in Angola.

President Habyarimana was shot down initiating the Hutu genocide perpetrated by the Tutsi. By July 3, 1994 - that is only about three months later - the Tutsi RPF overran the capital city of Kigale effectively ending the war. The absence of western intervention in this case is well known and usually discussed in the context of preventing the genocide. Given the timeline this was probably not feasible: the rapid forms of western intervention - air power, special forces - are ineffectual against large groups of people welding machetes, and by the time massive numbers of ground troops could have been put in place it would have been far too late. Rather the lack of western intervention is a case study in how prolonged war can be avoided: the Tutsi won and the peace of the strong over the weak has prevailed since.

We want to emphasize just how short are transitional wars compared to the prolonged conflict brought about by insufficiently strong outside intervention. The transitional war that brought peace to Rwanda lasted months. Moreover, the length of wars appears largely unrelated to whether they are civil wars: although most prolonged wars are civil, absent outside intervention they tend to be relatively short, if not so short as in Rwanda. Reaching farther back in history, the U.S. Civil War, bloody as it was, lasted only four years. World wars - in which outside intervention is not possible more or less by definition - also have been relatively short: four years in the case of World War I and six years in the case of World War II.

The overall point is that these transitional wars are short: less than a decade in length and often lasting only months. Hence although they are bloody, because they are short they are not necessarily more bloody than a cold peace that lasts many decades.

4.4. Cold Peace

There are two types of cold peace: a one-sided cold peace where weak intervention is matched against the strength of the opponent and a two sided cold peace where two outside intervenors stare eye-to-eye across a border. The former seems rare: one example is in Afghanistan, which has had varied intervention policies over the years since civil war begin in 1978. Initially the Soviet Union intervened and a prolonged war resulted until the Soviet withdrawal in 1988. What followed seems to have been a cold peace where intervention was matched against the strength of the opponent: the Taliban overran most of the country, but a small enclave remained under the control of the Northern League propped up by foreign support. This persisted until the strong US intervention in 2001 resulting in the prolonged war that continues to this day.

The eye-to-eye stare across the border is a more common form of cold peace. The classical example is the Iron Curtain, where military forces of the intervenors - the US on the West and Soviets on the East - sat for decades eye-to-eye in the literal sense. A more interesting case is in the Sinai where the intervention on both sides is by the US. Naturally we do not see US soldiers staring at each other eye to eye across the border, but the essential element of the Camp David accords was the promise of substantial military support (in the form of equipment and training) for both sides. We do note that in some places outside intervenors in the form of UN blue-helmets patrol the boundary - taking both sides in effect - but their job is merely to act as monitors - they have not even enough military strength to protect themselves should a shooting war break out.

4.5. Prolonged War

As we have indicated, this seems the least justifiable form of intervention. The only rationale we can think of is that a region poses a particular danger and hence the importance of keeping it weak offsets the bloody harm of prolonged war. Yet, if we look at the record, Vietnam, Sudan, Angola, Lebanon, Syria, and, not listed in the table, Libya do not appear to have ever presented any great danger to the intervening powers. It is interesting that while the US intervention in Vietnam is widely criticized outside the US, it seems to be so for mostly the wrong reasons. Surely there was nothing wrong with supporting the South, for, despite all the shortcomings of its government, there was no popular desire to be ruled by the equally bad or worse government in the North. Nor can there be much moral doubt about opposing the spread of communism: one need not look further than North Korea and Cuba - two of the most miserable places in the world - to see that. Nor is it clear why the direct involvement of the US is worse than indirect Russian involvement. From our point of view the US should be rather criticized for creating a prolonged and costly conflict by attempting to maintain a balance of power in the South.

4.6. Balkanization

Our theory indicates that balkanization is unstable - yet we do see balkanizations in fact. As indicated in some cases this is because the theory does not apply: in the actual Balkans, for example, difficult mountainous terrain makes it difficult for one side to win. Our Assumption 4 (c) that if $\phi_j > 0$ is such that $\phi_j \leq \phi_k$ for all k with $\phi_k > 0$ then $r^0(\phi_j, \phi_{-j}) = 0$ is violated since even the weakest society will have resistance to losing land. In this case there can be states where - with or without outside intervention - every society has positive resistance to losing land and hence the state is recurrent. Since positive resistance events do occur over long periods of time, this means that there will be recurrent conflicts. This seems a relatively good description of the Balkans, which has been Balkanized and in a more or less perpetual state of conflict since about 1200 BCE with the exception of periods of time when strong outside powers (Rome, the Ottomans) imposed peace. Each time, with the withdrawal of the outsiders low level periodic conflict seems to have more or less immediately resumed. Most recently large parts of the Balkans have been absorbed into the EU which may play much the same role as the Romans and Ottomans in bringing peace to the region - what will happen at some future time when that power should be withdrawn we suspect will be a resumption of the old divisions.

However: even when terrain is good - such as in the Middle East below - we do sometimes see balkanizations over long periods of time. This suggests that our theory is too simple.

First, let us reiterate the argument about the instability of balkanization: it is a simple argument. Suppose we have a balkanization in which the active societies are stable. By assumption the outsiders cannot intervene for both of the two weakest societies. Hence one of these two weakest must have zero resistance to losing land, and there is zero resistance to the strongest society winning the land. Such a change does not affect which are the two weakest societies: so collectively the two weakest societies always have zero resistance to losing a unit of land until one of them vanishes.

If there is a single outside intervenor this makes perfectly good sense - it is not very practical in a multi-way conflict to consistently intervene on behalf of two different clients. However: with more than one outside intervenor it may be that each intervenes consistently on behalf of a different client. Specifically: if outside intervention consistently takes place on behalf of both of the two weakest societies and is sufficiently strong as to give both positive resistance to losing land then it is easy to construct intervention policies that preserve the balkanization: as soon as one of the two weakest societies wins a unit of land intervention is withdrawn until it loses the land again. The key point is that we can have stability in a balkanization provided that there is more than one outside intervenor against the weak parties, but it is not so likely with just one.

The Middle East is the most obvious example of a region which is generally perceived as a balkanization. Never-the-less it can be usefully analyzed by breaking it down into two sub-regional conflicts: Israel versus Egypt in the South and Sunni versus Shiite in the North. In the South Israel and Egypt have intermittently fought from 1948 (the Arab-Israeli war listed in our table) to 1973 (the Yom Kippur war) after which peace negotiations began and ended with a further cold peace in 1979. This cold peace, enforced by strong US intervention on both sides takes the form of a treaty that has largely resulted in the cessation of bloodshed. This appears to be the most desirable form of cold peace.

The situation in the North is on the other hand a story of insufficiently strong intervention and bloody, prolonged wars. We have first the Lebanese civil war, then the Iran-Iraq war, then the conquest of Kuwait, followed by the liberation, the second Iraq war, and now by the Syrian civil war. As can be seen in our table some of these conflicts can be broken into separate regions which can be usefully analyzed by our methods.

The Kurds form a particularly interesting sub-case in the North. We have assumed that the weakest power is the most likely to lose land. But in a multi-lateral conflict the larger powers may be so focused on fighting each other that a smaller power is able to survive "in the shadows" so to speak. Originally the Kurds were able to occupy land as a consequence of the civil disorder and no fly zone that followed the Iraqi defeat in Kuwait: Saddam Hussein's Sunni forces were tied up with defeating the Shia near Basra and especially the marsh Arabs. Following second Iraq war politically the Sunni's and Shia's were more concerned with each other than with the Kurds - who also received limited US support, very limited on account of the alliance with Turkey: indeed the official US position has always been that Kurdistan should be part of a unified Syria.

Syria has high resistance to losing land, especially with Russian support. The other major party to the conflict in Iraq and Syria, the ISIS appear to be classical zealots, unstable, having no resistance to losing land and who consequently must either win quickly or vanish forever. As they have not won quickly we expect they will shortly vanish forever except perhaps as a rump group of stateless terrorists like their predecessor Al Quaeda. ISIS land, our model predicts, can go to either Syria/Iraq or to the Kurds. Then there are two possible scenarios:

(1) The US withdraws support for Kurds. In this case at some point Kurds will lose land to Syria and Iraq and Kurdistan will vanish;

(2) The US holds its support for Kurds. In this case our model predicts a balance of power between Kurds and Syria - but reality is a little different: owing to US concern with Turkey, sustained US support for Kurds is possible only if the Kurds compromise on their requests for an independent state and commit in advance to be part of a united Syria (like the Iraqi Kurds in Iraq) when ISIS is defeated. If this is the case - and we think it indeed is the more likely scenario - the outcome may be a balance of power within the context of particular states.

5. Hegemonies in History

5.1. Hegemonies are Common

Our theory says that absent outside intervention we should generally see hegemony. As outside intervention is possible only in smaller conflicts it follows that we should generally see hegemony. The idea of history being dominated by hegemonic states may seem a strange one, but with some important exceptions it is borne out by historical facts.[10] Take, for example, the largely geographically isolated region of China: bounded by jungles in the South, deserts on the West, cold arid wasteland in the North and the Pacific Ocean in the East. We find that during the 2,234 years beginning from when we have decent historical records in 221 BCE the area was ruled by a hegemonic state roughly 72% of the time, with five interregna. Less reliable records exist for the area of Egypt, but in the 1,617 years from 2686 BCE to the end of the new Kingdom in 1069 BCE we see hegemonic rule 87% of the time with two interregna. In Persia during the 1,201 years from 550 BCE to 651 CE we see hegemony 84% of the time with two interregna. England has been largely hegemonic within the geographically confined area of the island of Britain for 947 years from 1066 CE to the present. The Roman Empire ruled the Mediterranean area as a hegemony for 422 years from the advent of Augustus in 27 BCE to the permanent division into Eastern and Western Empires in 395 CE and the Eastern Roman Empire lasted an additional 429 years until the advent of the Caliphate in 814 CE. The Caliphate itself lasted 444 years until the Mongol invasion in 1258. After a 259 gap, the Ottoman Empire established a hegemony over the same general area for 304 years from the conquest of Egypt in 1517 CE to the Greek revolution in 1821 CE.

5.2. Hegemonies occur when outsiders are weak

While hegemonies are common in history, there are two glaring exceptions: except for brief periods neither the subcontinent of India nor, following the fall of the Western Roman Empire, the area of continental Europe were subject to a hegemonic state. According to our theory hegemonies will not persist when there are strong outsiders protected by geographical barriers. In the case of both continental Europe and India this is the case. In the case of Europe following the fall of Rome and up to around 1066 we have the continued interference of northerners - the Vikings especially were well protected by their own geography. Following 1066 we have the constant interference of England - also safe behind a water barrier: during this period we observe that England constantly

[10]See Levine and Modica (2012) for data and sources.

intervened in continental conflicts but always to support the weaker side, and eventually this policy of balance of power became explicit.[11] India also was subject to repeated invasion from central Asia - protected not by water but by difficult desert and mountain terrain.[12] Of course China too was subject to outside influence - particularly that of the Mongols. However, the relative size of the Mongolia is quite small relative to China - less than half a percent of the population - while the population of Scandinavia was about 5% that of continental Europe, that in central Asia about 5% that of India, while England was about 8% of continental Europe. These exceptions are in fact exactly what is predicted by our theory: we show that as outside influence grows the fraction of time hegemony will reign decreases.[13]

The role of England in maintaining a balance of power on the continent is well documented and notorious for its complete cynicism. From the rise of Spain following the discovery of America in 1492 through Brexit in 2016 British foreign policy has largely been aimed at preventing a hegemony over continental Europe. Many books (see for example Sheehan (1996)) have been written on the topic and few discussions of European history fail to remark upon the remarkable fact that Britain consistently changed sides in conflicts to support the losing side. Most dramatic perhaps is the shift to an alliance with France in 1904 in the face of the German threat - this after nearly 1000 years of historical enmity against the French culminating in what many consider to be the true first world war: the Napoleonic wars. Note that until the advent of the European Union and the fall of the Iron Curtain this policy was quite successful. The latest effort to break up the continental hegemony of the EU - Brexit - may be less successful: contrary to the predictions of its advocates it seems to have strengthened pro-European sentiment on the continent. As the severely negative economic consequences to Britain begin to take effect continental desire for succession from the union will probably be further diminished. While it is true that warfare, economic or otherwise, has costs to the attacker as well as the defender, victory generally requires inflicting greater loss on the opponent than on yourself.

[11] It is not completely correct to view England and Scandinavia as "outsiders" as at various time they had continental interests and conversely, but the key point is that they had a core area relatively safe from invasion. In a different direction Hoffman (2013) argues a role also for the Western Catholic church which in Europe acted as a balancing force much akin to to the outsiders of our model.

[12] The exact nature of the asymmetry in the physical geographical barrier is uncertain, but it is a fact that India has been invaded numerous times successfully from Central Asia, but there have been no successful conquests of Central Asia from India. Phil Hoffman in a private communication suggests that part of the answer may lie in the fact that the area of Central Asia is well suited for raising horses and India is not, and that horses play a central military role in conflict between Central Asia and India.

[13] Note that geographical factors matter in our argument only in so far as they give rise to outsiders who influence the evolution of the relationships between the other groups. An existing literature, including Diamond (1998), gives physical geography a direct role, arguing for example that the terrain of Western Europe is more defensible than that of China, hence less susceptible to hegemony. Besides this particular claim being challenged on physical grounds (Hoffman (2013)), such considerations have no bite in the Indian case. Incidentally: while this discussion includes only the area of Europe, Asia and North Africa, it should be borne in mind that until modern times 90% of the world population lived in this area.

6. Conclusion

The model we have studied sheds some light on the issues related to intervention and peace, in particular on the trade-off between having the contenders reaching peace as quickly as possible - which usually happens with the strong dominating the weak - and protecting the weak - which may prolong the conflict. We have further seen that if the goal of protecting the weak is predominant, then to minimize the costs of war intervention should be strong enough to avoid going back and forth between states where one part in turn is considerably stronger than the other, and reduce the war to what we have called a “cold peace” - which can be thought of as “border skirmishes”, and hopefully ends in reaching an unarmed negotiation stage.

However, much is left to understand. The great success story of peace is the de facto US occupation of Western Europe after World War II, its role in NATO and promoting the European Union and encouragement of European politicians, especially in France and Britain, that led to durable peace and democratic institutions. This was enormously costly, and this kind of peace - real peace - has lacked success elsewhere. Indeed US efforts at nation-building outside Western Europe and Japan has been an abysmal failure. The greatest success, ending the Israel-Arab conflict, has succeeded only in creating a cold peace propped up by continued and costly US intervention.

An important question is to understand why the U.S. was so successful in Europe and Japan and so unsuccessful elsewhere. Was it simply the willingness to commit resources on a massive scale - a huge war effort, military occupation on a giant scale, money poured into reconstruction? One may say it is the cold war - the willingness to actively support Europe to counteract the Soviets - but the cold war effort to intervene in Vietnam was as colossal a failure as the European intervention was a success. Certainly understanding the success in Europe and Japan and failure elsewhere is something that needs to be understood. If it is simply a matter of resources and willingness to spend them, then perhaps the US success in Europe holds no useful lesson for peace.

In a similar vein we may wish to study earlier successful and unsuccessful attempts at nation building: for example, the British legacy in India is a stable and relatively peaceful democracy. The French legacy in their colonies is poor - and the Belgians horrific. To understand success and failure here is to understand whether or not hegemony is a good idea. From the analysis here, however, it is reasonable to conclude that intervention to prevent hegemony needs to be strong enough (or weak enough) to bring about a cold peace - intervention that brings about prolonged war cannot be good from the point of view of peace.

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Appendix 1: Balance of Power Segments

Lemma 1. *In a recurrent j,k balance of power segment there are no states below the intervention thresholds, that is if $L_j(z) < \overline{L}_{jk}$, or $L_k(z) < \overline{L}_{kj}$, then z is not in a j,k balance of power segment.*

Proof. Consider j. Suppose first that $r_j^0(\phi_j(L_j(z)), \phi_k(L - L_j(z))) > 0$. Then there is zero resistance to j increasing land, but positive resistance to j decreasing land. Hence z could only be a left endpoint of a balance of power segment. If this is the case then at z' where j has one more unit of land there would have to be zero resistance to j losing a unit of land. This would imply that at $L_j(z)$ there is outside help, that is $L_j(z) = \overline{L}_{jk}$ - hence that in the segment $L_j \geq \overline{L}_{jk}$.

Next suppose that $r_j^0(\phi_j(L_j(z)), \phi_k(L - L_j(z))) = 0$. Then if z was part of a balance of power segment it should be $L_j(z) > \overline{L}_j$, where j has positive resistance to losing land. But then it cannot be $L_j(z) < \overline{L}_{jk}$ because below $\overline{L}_{jk}$ resistance of j decreases with its land. □

Lemma 2. *Very strong and very weak intervention on behalf of j against k are ineffective in the sense that the recurrent balance of power segments remain unchanged if we take $\overline{L}_{jk} = 0$.*

Proof. In the case of very weak intervention the resistances are the same with or without intervention so the balance of power segments are the same.

In the case of very strong intervention there cannot be a balance of power segment below $\overline{L}_{jk}$ with or without intervention and the signs of the resistances do not change because of intervention when j has the threshold level of land or above. □

Lemma 3. *There is a recurrent paired balance of power segment if and only if intervention is weak for both j,k in which case the balance of power segments are the two short segments.*

Proof. Suppose intervention is weak for both j and k. The two segments in the assertion, namely from $\overline{L}_{jk}$ to $\overline{L}_{jk}+1$ going to the right and from $\overline{L}_{kj}$ to $\overline{L}_{kj}+1$ going to the left form indeed a paired segment, directly from the definitions. From Lemma 1 we know that no state to the left of $\overline{L}_{jk}$ or to the right of $\overline{L}_{kj}$, so what we have to show is that states between $\overline{L}_{jk}+1$ and $\overline{L}_{kj}+1$ do not belong to segments ("between" refers as usual to our preferred visualization). By hypothesis $r_k = 0$ at $L_k = \overline{L}_{kj}+1$, and it will be zero up to some $\overline{L}_{kj}+\ell_k$; analogously, $r_j = 0$ from $\overline{L}_{jk}+1$ up to some $\overline{L}_{jk}+\ell_j$ (going left in our pictures). If $\overline{L}_{kj}+\ell_k$ and $\overline{L}_{jk}+\ell_j$ do not overlap they must be one unit of land apart (formally, if $\overline{L}_{kj}+\ell_k+\overline{L}_{jk}+\ell_j < L$ their sum must be exactly $L-1$), because at the state where one resistance becomes positive the other must be zero; thus from $\overline{L}_{jk}+\ell_j$ down to $\overline{L}_{jk}+1$ there is zero resistance, and from $\overline{L}_{jk}+\ell_j$ up-left to $\overline{L}_{kj}+1$ too; and in the "hole" between two adjacent states there is positive resistance both ways. Therefore in this case there is no segment between the two segments. Suppose now that If $\overline{L}_{kj}+\ell_k$ and $\overline{L}_{jk}+\ell_j$ do overlap. The only difference from the previous case is that instead of the hole there is an interval between the two segments where points are linked by two-way zero-resistance; from the left end point of the interval there is zero resistance down to $\overline{L}_{jk}+1$, and from the right and point analogously up to $\overline{L}_{kj}+1$. None of these states can belong to a segment, as before.

Conversely, suppose there is a paired balance of power segment. Let us call "walls" the extremes of the segments. At the left wall of the rightmost segment j's resistance is positive, then going down, at the right wall of the leftmost segment j's resistance is zero (since k has positive resistance); and at the left wall of the left segment j's resistance is positive again. Therefore here is where support for j occurs, that is the left wall of the left segment is $\overline{L}_{jk}$. Similarly the right wall of the right segment is $\overline{L}_{kj}$. Starting from there going down to the left wall of the same segment there is no support for j but still its resistance is positive there. If the right segment were not short, in its interior j's resistance would have to be zero, but it is again positive at the left wall, which is impossible given there is no external support in that range. Thus the right segment is indeed the short one in the assertion, which implies intervention in support of k is weak as asserted. Notice in passing that j's resistance would be positive at the right wall if it were not for intervention in favor of k. The argument for the left segment is the same. □

Lemma 4. *If intervention for j is weak and that for k is strong or there is no intervention for k there is a single short recurrent balance of power segment starting at $\overline{L}_{jk}$ and the intervention on behalf of k is ineffective.*

Proof. Existence of a short segment starting at $\overline{L}_{jk}$ follows as in Lemma 3 from weakness of intervention for j: at the threshold j has positive resistance (since the intervention is not very weak) and so k has zero, we can move only to the right; but when we move one unit to the right, by definition of weak intervention k has positive resistance, hence j has zero and we can move only to the left; hence as we can move back and forth only between $\overline{L}_{jk}$ and $\overline{L}_{jk}+1$ we have a short segment. Since we know (Lemma 1) that states on the left of $\overline{L}_{jk}$ cannot be part of a segment, to establish uniqueness we only need to look to the right $\overline{L}_{jk}+1$.

Suppose there is no intervention for k. At $\overline{L}_{jk}+1$ there is positive resistance by k, and increasing j's land, as long as this is the case there is zero resistance to going left one step; at some point k's resistance may become zero (so there is a hole there) and from then on to j's hegemony k's resistance must remain null. Thus we have no other segment.

Suppose now that intervention for k is strong. Then it must be the case that $\overline{L}_{kj}$ is to the right of $\overline{L}_{jk}+1$, for otherwise j's resistance at $\overline{L}_{jk}$ would be zero. From $\overline{L}_{jk}+1$ up to where $L_k=\overline{L}_{kj}$ we must have null resistance by j - since it is zero where $L_k=\overline{L}_{kj}$, going left j has less land and no support in the range. And to the right of $\overline{L}_{kj}$the situation is essentially as in the previous case: as we increase j's land, the system can move one step to the left without resistance until a possible "hole" after which j's resistance becomes positive, then right to j's hegemony without resistance. Again there is no segment in the range.

As we have just seen the two cases yield the same stable configuration, so that intervention for k is ineffective. □

Lemma 5. *If intervention for j is strong and there is no intervention for k there is no recurrent balance of power segment.*

Proof. By Lemma 1 no state below $\overline{L}_{jk}$ can belong to a segment; at $L_j = \overline{L}_{jk} + 1$ resistance by k is zero, and absent intervention in its favor k's resistance remains null until j reaches hegemony, by monotonicity. Conclusion follows. □

Lemma 6. *If intervention for both is strong then there is a single long recurrent balance of power segment from $\overline{L}_{jk}$ to $L - \overline{L}_{kj}$.*

Proof. Again by Lemma 1 states with $L_j < \overline{L}_{jk}$ or $L_k < \overline{L}_{kj}$ cannot be part of a segment. By hypothesis at $L_j = \overline{L}_{jk} + 1$ resistance by k is zero, hence it remains zero up to where $L_k = \overline{L}_{kj} + 1$, then becomes positive at $\overline{L}_{kj}$ (where necessarily j's resistance is null). Similarly, going left from $\overline{L}_{kj}$ by hypothesis j's resistance is zero where $L_k = \overline{L}_{kj} + 1$ and by monotonicity it remains zero until where $L_j = \overline{L}_{jk} + 1$, then it is positive at $L_j = \overline{L}_{jk}$ (where k's resistance is null). This is what we had to show. □

Appendix 2: Proof of the Dynamic Theorem

Lemma 7. *Hegemonies and balance of power segments with positive radius are recurring communicating classes of P_0.*

Proof. By definition there is positive resistance to leaving when the radius is positive and in the case of segments zero resistance to moving from any state in the segment to any other. Since positive resistance means zero probability in P_0 and zero resistance means positive probability in P_0 this is the definition of a recurring communicating class of P_0. □

The basin of a recurring communicating class S is the set of points B such that for z in B there is a zero resistance path to some state in S and no zero resistance path to any point in any other recurring communicating class. The *Ellison radius* of a recurring communicating class is the least resistance of paths out of the basin and is denoted by E_S.

Lemma 8. (No Balkanization) *If there are three or more societies there is zero resistance to reaching a binary state or a stable hegemony.*

Proof. We show that there is zero resistance to a configuration with one fewer active society. First, suppose that there is an unstable society. Regardless of intervention policy this society has zero resistance to losing a unit of land, hence zero resistance to losing all of its land. So suppose there are three or more active stable societies. Let j, k be two of the active societies with the least aggregate power. Then regardless of intervention by Assumption 4(c) at least one of these has zero resistance to losing land and there is zero resistance to a third society getting the land (Assumption 6(a)), hence zero resistance to the other society not receiving the land. Hence there is zero resistance to the total units of land of these two societies decreasing by one. If we are at a binary in a balance of power basin there is a zero resistance path to the limit set corresponding to the unique balance of power segment. Otherwise there is a zero resistance path to a hegemony. Either the hegemony is stable or the superzealots create a zero resistance path to a stable hegemony. □

Lemma 9. (Hegemonies go everywhere) *From a hegemony the least resistance of reaching a particular target state is always the hegemonic resistance regardless of the target and if the hegemonic resistance is positive this is the Ellison radius of the hegemonic state.*

Proof. Starting in hegemony the resistance to superzealots gaining a unit of land is the same as for any other society, the hegemonic resistance. Once the superzealots get a unit of land there is zero resistance to their establishing a hegemony. Since the superzealots are unstable they may with zero resistance lose land to any society stable or unstable hence reach any state with zero resistance. Hence there is a path from hegemony to any state with resistance equal to the hegemonic resistance and no path out of hegemony can have less resistance than this. □

Lemma 10. *A binary state with no recurrent balance of power segment (including binary states where one or both societies are unstable) has zero resistance to reaching hegemony.*

Proof. If there is an unstable society then it has zero resistance to losing land hence reaching a hegemony of the other society. Suppose then that both societies, say j and k, are stable. Suppose $r_k(z) > 0$,which implies that $r_j(z) = 0$; then going left j's resistance must remain null down to k's hegemony, otherwise the state when it would become positive would be the lower bound of a short recurrent segment. Analogous argument hods if $r_j(z) > 0$. If finally $r_j(z) = r_k(z) = 0$ then either to the left or to the right we must have no resistance to hegemony, otherwise z would be an interior point of a long recurrent segment. □

Lemma 11. *For a balance of power segment the Ellison radius is the radius and there is a path to hegemony with resistance equal to the radius.*

Proof. The only thing to be shown here is that from a segment it cannot have less resistance to getting out of the basin by having a third society enter than by having one of the two societies lose all its land. This follows directly from the assumption 6(b) that the resistance to an inactive society entering is higher than the least resistance to reach an hegemony. □

Lemma 12. *For $\epsilon > 0$ the system P_ϵ is ergodic and aperiodic and the only recurrent communicating classes in P_0 are the hegemonies and balance of power segments with positive radius (provided this set is not empty), all remaining states being transient.*

Proof. The system P_ϵ is aperiodic because at any state there is positive probability of staying (assumption $r_0(z) = 0$ all z) and positive probability of leaving (assumption $r_j(z) < \infty$ all z).

Since all resistances are assumed finite, from any state there is a positive probability of reaching a hegemony: one active society keeps gaining land until it has it all. From Lemma 3 there is positive probability of going anywhere from a hegemony hence a positive probability of reaching any state from any other state. This gives ergodicity.

In P_0 a hegemony with zero hegemonic resistance is transient since by Lemma 9 it has positive probability of reaching a recurrent communicating class.

In a binary state not in a balance of power segment there is zero resistance to either reaching hegemony or a balance of power segment. The latter is a recurring communicating class and the former is either a recurring communicating class or transient, hence such points are transient.

Finally in a Balkanization by Lemma 8 there is a zero resistance path to a binary state which we already showed is either a recurring communicating class or transient, hence Balkanizations are transient. □

Corollary 1. *For all recurring communicating classes the Ellison radius is the radius.*

Proof. Just summarizes results proved in the Lemmas. □

Following Levine and Modica (2016) we say that a collection of recurrent communicating classes $\mathcal{C}$ forms a *circuit* if for any pair $C, C' \in \mathcal{C}$ there is a sequence $C_1 = C, C_2, \ldots, C_n = C'$ with $C_i \in \mathcal{C}$ such that the transition from C_i to C_{i+1} along the sequence has resistance equal to the radius of C_i. In words, any pair of elements in the circuit are linked by a least-resistance path within the circuit. A circuit is maximal if there is no larger circuit that contains it. *A super-circuits* consists of a circuit of circuits where resistance between one circuit and another is measured by minimizing over pairs of recurrent communicating classes in source and target circuit the difference between the least resistance from the source to the target and the radius of the source.

Corollary 2. *(a) Each paired balance of power segments with modified radius greater than the radius forms a maximal circuit.*

(b) The hegemonies with positive radius, single balance of power segments and paired balance of power segments with modified radius equal to the radius (that is all recurring communicating classes with modified radius equal to the radius) form a single maximal circuit.

(c) There is a single super-circuit in which the modified radii defined in Levine and Modica (2016) are the modified radii.

Proof. (a) By definition there is less resistance to going to the paired segment than to hegemony - hence the pair forms a maximal circuit.

(b) All the described recurring communicating classes are either hegemonies or reach any hegemony on a path with resistance equal to the radius (by Lemma 9), hence all have a least resistance path to any state at all. The circuit is maximal by part (a) as the paired circuits are not also part of the single circuit which consists of all the states not in the paired circuits.

(c) That there is a single super-circuit follows from the fact that the paired circuits all be connected to hegemonies by second least resistance paths and hegemonies can go anywhere. The modified radius was defined exactly as in Levine and Modica (2016). □

Theorem (Theorem 2 in the text). *(1) When $\epsilon = 0$ states with $R_z = 0$ are transient, states with $R_z > 0$ are positively recurrent, states with $R_z > 0$ that are hegemonic are absorbing and balance of power segments with $R_z > 0$ are absorbing but starting from any state in the segment all other states in it are hit infinitely often.*

(2) When $\epsilon > 0$ there is a unique ergodic distribution μ_ϵ with a unique limit $\mu_0 = \lim_{\epsilon \to 0} \mu_\epsilon$ and $\mu_0 = 0$ if $R_z = 0$. If $R_z, R_x > 0$ then

$$0 < \lim_{\epsilon \to 0} \frac{\mu_\epsilon(z)}{\mu_\epsilon(x)} \cdot \frac{\epsilon^{M_x}}{\epsilon^{M_z}} < \infty$$

and there are constants $0 < C < 1 < D < \infty$ such that starting at z the expected hitting time T before reaching a different hegemony or balance of power segment with $R_B > 0$ satisfies

$$C\epsilon^{-R_z} \leq T \leq D\epsilon^{-R_z}.$$

The actual escape when it occurs is short and has expected length bounded above by D.

Proof. (1) Lemma 6.

(2) First part: Corollary 2 and Theorem 10 in Levine and Modica (2016).

Second part for $R_z > 0$: Corollary 1 and Theorem 4 in Levine and Modica (2016), for $R_z = 0$ Corollary 1 and Theorem 1 in Levine and Modica (2016). □

Appendix 3: Optimal Intervention

Suppose that the intervenor has a fixed level of power φ_0 and wishes to create the greatest level of stability in a balance of power between j, k - that is to create a segment with maximum radius. What intervention policy should it choose? The answer is this:

Proposition 1. *Optimal intervention has $\overline{L}_{jk}$ and $\overline{L}_{kj}$ at most one unit of land apart. Among all short segments formed by such pairs, the optimal ones (there can be one or two of them) are those with the maximum radius.*[14]

Proof. Consider one such pair, and suppose $R_z = R^\ell_{jk}$ that is k's hegemony is the easier to reach. Then lowering $\overline{L}_{kj}$ (that is pushing it to the right) has no effect on radius; and lowering $\overline{L}_{jk}$ (weakly) reduces the radius by monotonicity. So no single segment can improve upon the given choice. Now as we increase L_j: resistance to k's hegemony will be lower than to j's hegemony up to a point where it becomes higher. The candidates maximum radius segments are the two formed at the tilting point, so the maximum can be reached by one of them or by both. It remains to be checked that a paired segment cannot do better than the best single segment. In a paired segment created by (weak/weak) intervention, as in any paired segment, for one of them - say the left one - the radius is the resistance to reaching hegemony; that is, $0 < R^\ell_{jk} - R^{rr}_{jk} < R^r_{jk} - R^{\ell\ell}_{jk}$, and in the left segment $M_z = R^\ell_{jk}$, and in the right one $M_z = R^{\ell\ell}_{jk} + R^\ell_{jk} - R^{rr}_{jk}$. The inequality implies that in the right segment $M_z < R^r_{jk}$; therefore the single segment having as bounds (and intervention thresholds) the two extremes of the paired segment fares weakly better than that. □

[14]We must include segments of length zero here. If we wanted to keep with the notation in the text we should assume L odd, in which case the result is that $\bar{L}_{jk}$ and $\bar{L}_{kj}$ are exactly one unit of land apart.