1. Again Crossroad, with different numbers for a change. Actions as always Pass, Not Pass, payoff matrix

	P	S
P	-1, -1	1.5, 0
S	0, 1.5	1, 1

We know the Nash equilibria of this game: the two asymmetric off-diagonals, plus the symmetric mixed equilibrium which is easily computed to be $p = q = 1/3$. Payoff in this equilibrium is the same for both players, and as you can check it is 2/3.

Let us see what we can do with correlated equilibrium. We want to avoid the *PP* profile, so we set up a random device with three outcomes $\{x, y, z\}$ and probabilities $\{p, q, r\}$ which we want to distribute to the three profiles we want to play. Picture:

	$z; r$
$x; p$	$y; q$

We have to determine values p, q (being $r = 1 - p - q$) and an information structure to get a correlated equilibrium. We try with the same kind of informational structure we have used for the battle of sexes, which is *canonical* in that the partition cells correspond to players' actions. For player 1 $\{\{z\}, \{x, y\}\}$, for player 2 $\{x, \{y, z\}\}$. Then equilibrium has each player follow recommendation, that is play the corresponding action. Recall that players compute conditional probabilities where appropriate. Let us see what the no-deviation property entails. For player 1, if he is told z he should play P which is as good as it can get; if he is told $\{x, y\}$ then conditional probabilities are $p/(p+q), q/(p+q)$ and he should (weakly) prefer S to P assuming 2 follows recommendation, that is it must be

$$1 \cdot \frac{q}{p+q} \geq (-1) \cdot \frac{p}{p+q} + \frac{3}{2} \cdot \frac{q}{p+q} \quad \text{that is} \quad q \leq 2p$$

For player 2 analogously we should check that if she is told $\{y, z\}$ she should prefer S to P that is

$$1 \cdot \frac{q}{q+r} \geq (-1) \cdot \frac{r}{q+r} + \frac{3}{2} \cdot \frac{q}{q+r} \quad \text{that is} \quad q \leq 2r$$

Thus it must be $q \leq 2p, 2r$ with $r = 1 - p - q$. This says $\frac{1}{2}q \leq p \leq 1 - \frac{3}{2}q$ which implies $q \leq \frac{1}{2}$. Now look for a symmetric equilibrium. Equality of the player's payoff is

$$q + \frac{3}{2}r = q + \frac{3}{2}p \iff p = r$$

which since $r = 1 - p - q$ implies $q = 1 - 2p$; and from this and $q \leq \frac{1}{2}$ we get $p \geq 1/4$. Since payoff is $q + \frac{3}{2}p = 1 - \frac{1}{2}p$ we want p as small as possible - that is $p = 1/4$. The conclusion is that the correlated equilibrium which gives maximum equal payoff to the two players is

$$p = \frac{1}{4}, q = \frac{1}{2}, r = \frac{1}{4} \quad \text{with payoff} \quad 1 - \frac{1}{2}p = \frac{7}{8} = \frac{21}{24}$$

Picture:

	1/4
1/4	1/2

Compare correlated payoff with Nash payoff $\frac{2}{3} = \frac{16}{24}$: it is almost 30% higher.

2. Moulin-Vial Example

$$\begin{pmatrix} 0,0 & 4,2 & 2,4 \\ 2,4 & 0,0 & 4,2 \\ 4,2 & 2,4 & 0,0 \end{pmatrix}$$

Unique NE uniform, payoff 2 each. Correlated equilibrium yielding payoff 3 each with canonical device

$$\begin{pmatrix} 0 & 1/6 & 1/6 \\ 1/6 & 0 & 1/6 \\ 1/6 & 1/6 & 0 \end{pmatrix}$$

Solution

NE equilibrium must be symmetric. Letting $(p, q, 1 - p - q)$ 2's mixture 1 must have

$$\begin{cases} 4q + 2(1 - p - q) = 2p + 4(1 - p - q) \\ 4q + 2(1 - p - q) = 4p + 2q \end{cases}$$

from which $p = q = 1/3$ follows directly. For correlated equilibrium just check that deviating is not strictly preferred to obeying.