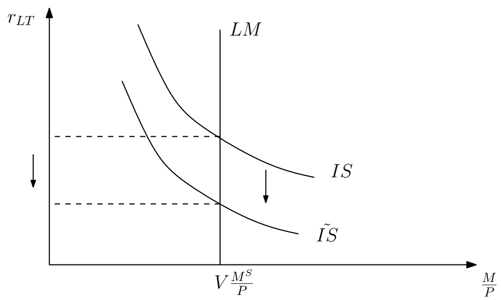
Keynes vs Classical Quantity Theory: A view from Louis Makowski[1]

We have assumed that demand for real balances is given by $L^{\text{tr}}(Y, i) = Y/V(r)$ with V increasing in r, the justification being that as the opportunity cost of holding money increases its demand falls.[2] And we talked about *transaction* demand (liquidity needed for daily use), which we emphasize now by writing L^{tr}. Of course, even under fixed P, if V were constant - that is $L^{\text{tr}} = kY$ - then the whole Keynesian argument about the usefulness of government intervention would collapse because if V is constant the LM is vertical so any shift in the IS curve is ineffective and the only meaningful policy instrument is money supply - as asserted by Classical Quantity Theory (CQT). Figure 0.1 illustrates. Notice that P is fixed here, the problem lies in vertical LM. A negative demand shock (real side) is absorbed by a fall in the interest rate (money side). No effect on Y. Still, real shocks did cause the Great Depression. How? As is clear from he picture, if M^S/P does not move the only way LM can shift inwards is if V goes down. Does V go down following a real shock? Historical data now available show that velocity does indeed decrease in recessions [CHART HERE], but as it happens Keynes agreed with the monetarists that transaction demand for liquidity is actually not significantly sensitive to interest rate, hence that it is appropriate to assume $L^{\text{tr}} = kY$. What then?

Figure 0.1: CQT straitjacket



Note the subscript in r_{LT} in the figure, it means long term. r_{LT} is the interest on perpetuities, that is the price of an income stream of γ from next year forever is $P_b = \gamma/r_{LT}$. In the investment-savings market lending is long term. Keynes argued that r_{LT} may not decrease as much as needed to keep Y from falling, because at low enough values lenders will expect the rate to go up hence they will abstain from lending and wait for better rates; thus they will temporarily keep

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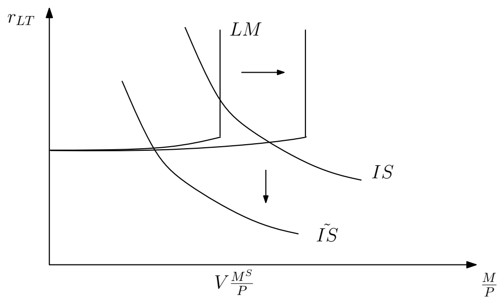
[2]We are neglecting inflation for now: assume $\pi = 0$.

their funds in cash balances, that is demand liquidity for *speculative* motives; if Y is low so will $L^{\mathrm{tr}}(Y)$, but at low enough rates lenders are willing to absorb all of $M^S/P - L^{\mathrm{tr}}(Y)$ in exchange for existing bonds, so that $L^{\mathrm{tr}} + L^{\mathrm{sp}} = M^S/P$, with obvious notation for L^{sp}. Hence the LM curve will bend inwards, much as in figure 0.2, and a severe real shock will cause a recession. Moreover, monetary policy cannot do much to help since this shape of the LM leads directly into a liquidity trap, see the figure. The present note reproduces Louis Makoswki's review of the argument, from lecture notes of him.

Figure 0.2: Liquidity trap recession



Rates of interest

We have so far assumed that wealth can be held in bank deposits paying no interest - which we call D - or bonds b paying r_{LT} (long term lending); here we add an intermediate asset T, time deposits (short term lending) paying interest r_{ST}. Assume lenders are risk neutral, so that their expected utility is equal to expected value. And assume they hold the same expectations: let r_{LT}^e the commonly expected long term rate next year, so that $P_b^e = \gamma/r_{LT}^e$ is the bond's expected price.[3] Consider a lender's choice between lending short term versus long term. If she buys a bond at price P_b, next year she will get $\gamma + P_b^e$ (coupon plus expected resale value of the bond); if she lends the same money short time she gets $(1+r_{ST})P_b$. Unless these two values are the same, her demands for bonds would be zero (P_b too high) or infinite (P_b too low).[4] Thus in equilibrium it must be $(1 + r_{ST})P_b = \gamma + P_b^e$, that is (check the algebra)

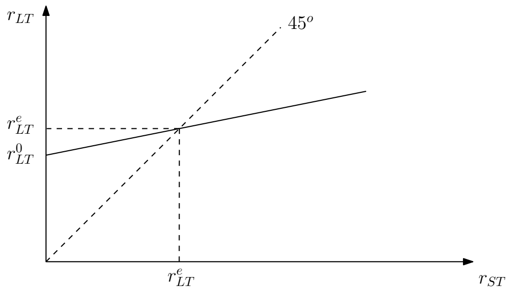
$$r_{LT} = \frac{r_{LT}^e}{1 + r_{LT}^e}(1 + r_{ST}).$$

[3]We are assuming that r_{LT}^e does not depend on r_{LT}, that is "inelastic expectations".

[4]Here we are using risk neutrality. Without it we should compare expected utilities, not expected values.

This establishes the one-to-one relation between r_{LT} and r_{ST} drawn in figure 0.3. Notice that the two rates are equal when they are both equal to r^e_{LT}. Note also the so called "term structure" of interest rates in this simple setting: if $r_{LT} < r^e_{LT}$ then $r_{ST} < r_{LT}$ (the usual relation); if $r_{LT} > r^e_{LT}$ then $r_{ST} > r_{LT}$ (in this case lending long term has low return because $r_{LT} > r^e_{LT}$ means bond price is expected to increase, that is a capital gain is expected).

Figure 0.3: Relation between r_{LT} and r_{ST}



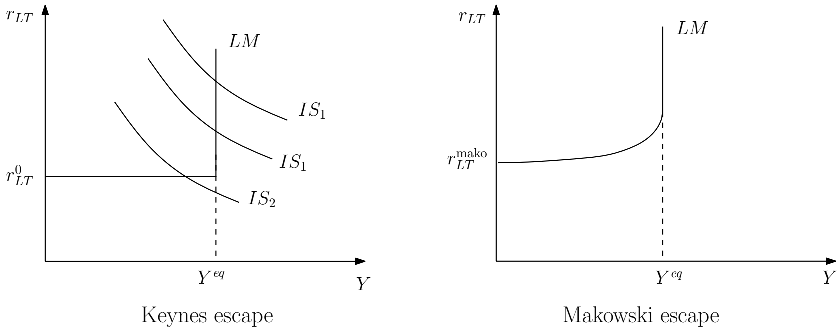
JMK escape from CQT: $r_{ST} = 0$.

As long as $r_{ST} > 0$, that is $r_{LT} > r^0_{LT}$, lenders leave no idle balances in deposits, so all liquidity demand is transactional hence independent of interest, therefore the LM curve is vertical. But independently of the level of Y, when $r_{ST} = 0$, that is $r_{LT} = r^0_{LT}$, lenders are indifferent between lending long term and keeping money in deposits, so the argument for $L^{\mathrm{tr}} + L^{\mathrm{sp}} \geq M^S/P$ goes through. This means that the LM will be flat at r^0_{LT}, as in figure 0.4, left panel (we are drawing the vertical part at $Y = Y^{eq}$ now for emphasis). A modest real shock will be absorbed by downward movement of the interest rate(s), but a large one will lead to a liquidity trap recession.

Credit crunch recessions, fast money and velocity

Real shocks are not the only causes of recessions. A shrinking of money supply is another one: the vertical part of the LM (be it a part or all of it) shifts leftwards, producing a fall in income and a *rise* in interest rates (see one of the figures above displaying the LM). We start from the case of vertical LM, where all money demand is transactional hence independent of interest rate. What happens to velocity of money? In the CQT case we know V remains constant and the LM contracts in proportion to money supply ($Y = V \cdot (M^S/P)$, see figure 0.1). But we know that V decreases during recessions. Can we explain that?

Figure 0.4: Escape from CQT



Think of liquidity we regularly keep in D. What do we need it for? Essentially for our daily little expenses, and little else. If we buy a car we get the needed money out of T at the last minute, and the minute after the car seller gets it she puts it in her T. The same holds for consumer durables (the family investments), and of course for the firms investment goods. The time the liquidity used for these "big ticket" transactions lies idle is virtually zero; and this means its velocity (which is 1/idle-time) is virtually infinite.[5] Makowski calls them *fast money transactions.* These do not affect transaction demand for liquidity. The only transactions we need cash for are the slow money transactions we make at food shops and the like. That is, transaction demand for money does not depend on all $Y = C + I$, but only on C - where we mean non-durable goods, for durables go into I. Formally, once we account for fast money we get

$$L^{\text{tr}} = kC.$$

We can as usually assume that $C = C_a + cY$, where $C_a > 0$ is consumption expenditure we try to sustain independently of income fluctuations, and $c < 1$ is the marginal propensity to consume out of Y. If we now write the LM we get $k(C_a + cY) = M^S/P$, which after rearranging can be written as

$$Y = \frac{M^S}{P} \cdot \left[\frac{1}{ck} - \frac{C_a}{c\frac{M^S}{P}} \right].$$

This says that velocity, the term in square brackets in the above expression, *goes down* under a monetary contraction - making the credit crunch recession harsher than if velocity were constant. Mathematically this is trivial: if $M^S/P = kY$ then $\Delta Y = \frac{1}{k}\Delta\frac{M^S}{P}$; but if $M^S/P = kC = k(C_a + cY)$ then $\Delta Y = \frac{1}{ck}\Delta\frac{M^S}{P}$ - of much larger magnitude if c is small. More interesting is the economics behind it. In a

[5] Observe that the same holds in fact also for all monthly bills.

recession the future becomes darker, and big ticket transactions (investments by families and firms) get postponed, much more than daily consumption - as is well documented empirically. Hence during a recession the economy mainly loses its fast money transactions, and money gets relatively more involved in slow money exchanges; therefore velocity - an average over all transactions - decreases.[6]

Makowski escape from CQT: $r_{ST} > 0$?

Now we can include interest rates in the fast money model. We focus again on real shocks, refer to figure 0.4. Is $r_{ST} = 0$ a necessity in a liquidity trap? Let us first reconsider Keynes argument for the fall of r_{ST} to zero in recessions. Starting from income equal to Y^{eq} suppose it falls to some value $Y < Y^{eq}$. Money market was in equilibrium, so that $M^S/P = L^{\text{tr}}(Y^{eq}) = kY^{eq}$; now excess money supply equals $M^S/P - L^{\text{tr}}(Y) = L^{\text{tr}}(Y^{eq}) - L^{\text{tr}}(Y) = k(Y^{eq} - Y)$. These are idle balances people no longer need for transactions, so if $r_{ST} > 0$ they lend short term and the liquidity imbalance persists. To induce people to keep them in D as speculative balances the short term rate must fall all the way to zero. With fast money the situation is analogue but the order of magnitude of idle balances is smaller. Indeed, now $L^{\text{tr}} = L^{\text{tr}}(C) = kC = k(C_a + cY)$, in equilibrium it was $M^S/P = L^{\text{tr}}(C^{eq})$ so excess supply is now $L^{\text{tr}}(C^{eq}) - L^{\text{tr}}(C) = ck(Y^{eq} - Y)$, that is c times what it was before. Can there be a reason to hold these more limited balances idle other than for a speculative motive? Makowski argues that if $Y < Y^{eq}$ firms may want to hold idle balances even if $r_{ST} > 0$ for a *precautionary* motive.

The idea of precautionary balances is simple: in a recession the big problem is getting paid. If your customers don't pay you then you can't pay your suppliers, then they won't be able to pay theirs, and so on - unless all have precautionary balances in D. Cautious firms will want to hold precautionary balances during a recession to avoid cash flow problems. We call this liquidity demand $L^{\text{prec}} = L^{\text{prec}}(Y^{eq} - Y, r_{LT})$, increasing in the first argument (severity of recession) and decreasing in the second (liquidity cost).

Observing that $C = C_a + cY^{eq} - c(Y^{eq} - Y)$ we can write total liquidity demand as

$$L^{\text{tr}} + L^{\text{prec}} = kC_a + ckY^{eq} - ck(Y^{eq} - Y) + L^{\text{prec}}.$$

Precautionary demand at Y^{eq} is negligible, so we may assume that money supply equals transaction demand at Y^{eq}: $M^S/P = kC_a + ckY^{eq}$. So, recalling that $ck(Y^{eq} - Y) = L^{\text{tr}}(C^{eq}) - L^{\text{tr}}(C)$, we end up with

$$L^{\text{tr}} + L^{\text{prec}} = M^S/P + L^{\text{prec}} - [L^{\text{tr}}(C^{eq}) - L^{\text{tr}}(C)].$$

[6] The argument relies on the fact that $C_a > 0$. Otherwise the fall in income is still $\frac{1}{ck}\Delta\frac{M^S}{P}$ but velocity is constant.

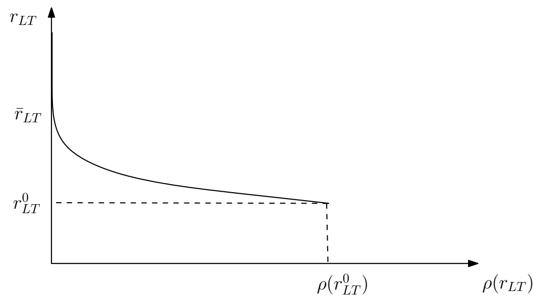
Hence the condition that total demand absorbs supply $L^{\rm tr} + L^{\rm prec} \geq M^S/P$ is $L^{\rm prec} \geq [L^{\rm tr}(C^{eq}) - L^{\rm tr}(C)]$: precautionary demand must cover the fall in transaction demand in recessions, when $Y < Y^{eq}$. This surely happens at r^0_{LT}, that is when liquidity has zero cost ($r_{ST} = 0$). Is it reasonable to suppose that the same occurs when liquidity cost is small but positive? $L^{\rm tr}(C^{eq}) - L^{\rm tr}(C)$ is the amount of idle balances at Y with fast money. So the vertical intercept of the LM is the rate at which firms are willing to keep those balances for a precautionary motive. We are talking of the amount of balances which consumers hold for daily needs, and it seems of the same order of magnitude of the potential cash flow problem (which after all starts from end consumers). Thus the condition does not seem particularly far fetched.

We now specify a functional form of $L^{\rm prec}$ to obtain the LM curve, $L^{\rm tr} + L^{\rm prec} = M^S/P$. Specifically, we will assume that it has the form

$$L^{\rm prec}(Y, r_{LT}) = (Y^{eq} - Y + \epsilon) \cdot \rho(r_{LT})$$

where $\epsilon > 0$ is a small number and ρ is smooth decreasing, and zero for $r_{LT} \geq \bar{r}_{LT}$, as in figure 0.5.

Figure 0.5: The function ρ



Thus $L^{\rm prec} = 0$ if r_{LT} is too high, but if not it will be positive (though small) even if $Y = Y^{eq}$. This is crucial for what follows.[7] If $Y < Y^{eq}$ then precautionary balances are increasing in the severity of recession and decreasing in their opportunity cost r_{LT}.[8]

Total liquidity demand $L^{\rm tr} + L^{\rm prec}$ must increase in Y for any $r_{LT} > r^0_{LT}$, where for $r_{LT} = r^0_{LT}$ (that is $r_{ST} = 0$) we may admit that it is independent of income. Since $\partial(L^{\rm tr} + L^{\rm prec})/\partial Y = ck - \rho(r_{LT})$ we then assume $\rho(r_{LT}) < ck$ for $r_{LT} > r^0_{LT}$

[7] We are assuming that everything is deterministic but obviously the world is more complicated than that. A little precaution is justified as a reaction to uncertainty.

[8] Remember that between r_{LT} and r_{ST} there is a one-to-one correspondence so it is irrelevant which one we use as function argument.

and $\rho(r^0_{LT}) \leq ck$. On the other hand the condition $L^{\text{prec}} \geq [L^{\text{tr}}(C^{eq}) - L^{\text{tr}}(C)]$ reads

$$(Y^{eq} - Y + \epsilon) \cdot \rho(r_{LT}) \geq ck(Y^{eq} - Y) \iff \rho(r_{LT}) \geq ck\frac{Y^{eq} - Y}{Y^{eq} - Y + \epsilon}.$$

The assumption that this holds for any $\epsilon > 0$ at r^0_{LT} then becomes $\rho(r^0_{LT}) \geq ck$. Thus we impose $\rho(r^0_{LT}) = ck$.

Now the LM is $L^{\text{tr}} + L^{\text{prec}} = M^S/P$, that is $\rho(r_{LT}) = ck\frac{Y^{eq}-Y}{Y^{eq}-Y+\epsilon}$. For $r_{LT} \geq \bar{r}_{LT}$ we have $\rho = 0$ and the equality implies $Y = Y^{eq}$; thus above $\bar{r}_{LT}$ the LM is vertical.[9] At the other end, for $Y = 0$, from $\rho(r^0_{LT}) = ck$ it follows that $\rho(r^0_{LT}) > ck\frac{Y^{eq}}{Y^{eq}+\epsilon}$ which says that at $Y = 0$ it is $L^{\text{tr}} + L^{\text{prec}} > M^S/P$ if $r_{LT} = r^0_{LT}$; so the vertical intercept of LM, given by $\rho(r_{LT}) = ck\frac{Y^{eq}}{Y^{eq}+\epsilon}$, is at some $r^{\text{mako}}_{LT} > r^0_{LT}$. We also know that $r^{\text{mako}}_{LT} < \bar{r}_{LT}$ because $\rho(r^{\text{mako}}_{LT}) > 0$. It remains to see what happens in the region of the (Y, r_{LT}) plane with $0 < Y < Y^{eq}$ and $r^{\text{mako}}_{LT} < r_{LT} < \bar{r}_{LT}$; notice that for such r_{LT} we have $\rho(r_{LT}) < \rho(r^{\text{mako}}_{LT}) < ck$. Re-writing the LM condition as

$$(Y^{eq} - Y)[ck - \rho(r_{LT})] = \epsilon\rho(r_{LT})$$

and differentiating we obtain

$$-[ck - \rho(r_{LT})]dY - (Y^{eq} - Y)\rho'(r_{LT})dr_{LT} = \epsilon\rho'(r_{LT})dr_{LT}$$

whence

$$\frac{dr_{LT}}{dY} = -\frac{1}{\rho'(r_{LT})}\frac{ck - \rho(r_{LT})}{[\epsilon + (Y^{eq} - Y)]} > 0$$

so the LM is increasing in Y from zero to Y^{eq}. We have drawn it in figure 0.4, right panel. Makowski's LM bends backwards gently and stays higher than Keynes's.

[9] If money supply changes the vertical part shifts correspondingly.