Zero lower bound with shock-dependent target

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Abstract

In a standard *IS-MP-PC* model where the natural real interest rate depends on demand conditions, if the Taylor rule contains the central bank's estimate of the equilibrium rate, then after an adverse demand shock the "natural" income level can be restored by monetary policy, that is the zero lower bound is not binding, if and only if the after-shock equilibrium nominal rate at the target inflation rate is not too low. Assuming that the central bank learns the after-shock equilibrium real rate over time implies that when it restores the desired income level it also brings inflation back at the target level, by expanding the AD curve.

1. Introduction. As Bernanke (2017) recently put it, "The risk of hitting the zero lower bound depends importantly on the "normal" level of interest rates, that is, the level of rates expected to prevail when the economy is operating at full employment with price stability and monetary policy is at a neutral setting." In *IS-MP-PC* models this translates into the fact that the zero lower bound (ZLB) line is higher - hence more easily hit - the lower the equilibrium nominal rate (see for example Buttet and Roy (2015)). In this note we present a Taylor rule where as advocated by Woodford (2001) the equilibrium real rate depends on demand conditions; and we take the further step of assuming that the rate appearing in the rule is the bank's estimate of the equilibrium rate - for the stress on estimates is pervasive in all discussions about policy.[1] In this model an adverse *permanent* demand shock makes the AD curve contract because the bank's estimate becomes too high. When the bank learns the new demand conditions and updates its estimate the AD goes back up - but it hits the higher ZLB sooner. Whether the bank is then capable of restoring the equilibrium "natural" income level (which is independent of the demand conditions) depends on the value of the equilibrium nominal rate in the new situation at the bank's inflation target. We find that below a *positive* threshold the bank cannot restore equilibrium, while if the shock changes the

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[1] See for example the discussions within the FOMC cited by Asso et al. (2010).

initial rate little enough it can. In an intermediate range the situation may be favorable or not depending on the slope of the PC curve.

That the bank learns the level of the after-shock equilibrium real rate implies that whenever the bank is able to restore the desired income level it also, at the same time, brings inflation back at its target level because, for example in the case of an adverse shock, a lowered estimated rate triggers a revision of the policy rule itself, which becomes more expansionary and drives the AD curve up - unlike for example in Weerapana (2013) and Groth (2016). To expand a little on this point: in the cited models (and the others we know) avoiding the liquidity trap means restoring the desired output level, at the cost of missing the inflation target; in the learning model the issue becomes when the central bank can restore both the desired output level and the target inflation.

2. The model and equilibrium. Time is discrete, and inflation is defined as

$$\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}.$$

The supply side of the economy is represented by the Phillips curve

$$\pi_t = \pi_t^e + \delta(y_t - y^{eq}) \tag{PC}$$

where π_t^e is the expected inflation and y^{eq} is the equilibrium log income as determined by the labor market, or by some long run trend. It results from the familiar AS curve by taking logs and subtracting the log of P_{t-1}. Observe that it is a line through the point (y^{eq}, π_t^e) in the (y, π) plane, with positive slope.

Inflation expectations are adaptive:

$$\pi_t^e = \pi_{t-1}$$

where π_{t-1} is the inflation rate observed at $t-1$. With adaptive expectations the PC curve at t goes through (y^{eq}, π_{t-1}); this implies in particular that, going from period t to period $t+1$, the curve shifts up if $\pi_t > \pi_{t-1}$ and down if $\pi_t < \pi_{t-1}$. Notice also that (PC) implies that inflation is constant if and only if $y_t = y^{eq}$.

The real rate of interest is defined as

$$r_t = i_t - \pi_{t+1}^e$$

and from $\pi_{t+1}^e = \pi_t$ we get $r_t = i_t - \pi_t$.

Turning to the demand side, essentially an *IS-LM* framework in the presence of inflation, we take a linear *IS* in log income y:

$$y_t = \mu - \beta r_t = \mu - \beta(i_t - \pi_t) \tag{IS}$$

where $\beta > 0$ will be constant and the demand conditions will be parametrized by μ. We assume $\delta < 1/\beta$ to avoid existence problems later. The equilibrium real rate $r^{eq}(\mu)$ is defined by

$$y^{eq} = \mu - \beta r^{eq}(\mu).$$

Now the Taylor rule, where we follow Woodford's (2001) recommendation of considering the above demand-dependent rate and we take the further step of inserting in the rule the bank's estimate of that rate. Denote by π^* the target inflation rate and let $\hat{r}^{eq} = r^{eq}(\mu) + \eta$ be the central bank's *estimate* of the equilibrium real rate, where η is an estimation error. We represent the monetary policy by the following interest-targeting rule:

$$i_t = \max\{0, r^{eq}(\mu) + \eta + \pi_t + a(\pi_t - \pi^*) + b(y_t - y^{eq})\} \tag{MP}$$

where a and b are positive coefficients. The idea is that the central bank regulates the money supply so that the money market is in equilibrium at the target interest rate. We will see shortly that effectiveness of monetary policy is determined by the sign of the equilibrium *nominal* rate at (μ, π^*) defined as

$$i^{eq}(\mu, \pi^*) = r^{eq}(\mu) + \pi^*.$$

For now observe that

$$r^{eq}(\mu) + \eta + \pi_t + a(\pi_t - \pi^*) + b(y_t - y^{eq}) = i^{eq}(\mu, \pi^*) + \eta + (a+1)(\pi_t - \pi^*) + b(y_t - y^{eq})$$

therefore if the bank's estimate is correct, that is $\eta = 0$, then at $(y_t, \pi_t) = (y^{eq}, \pi^*)$ we have $i_t \geq 0$ if and only if $i^{eq}(\mu, \pi^*) \geq 0$. If this is the case then at (y^{eq}, π^*) the error-free nominal target is such that the real rate $i_t - \pi^* = r^{eq}(\mu)$, the equilibrium rate. The aggregate demand curve *AD* usually obtained from the *IS-LM* intersection is in this framework derived from (IS) and (MP). Start by defining the zero-lower-bound line in (y, π) space

$$r^{eq}(\mu) + \eta + \pi_t + a(\pi_t - \pi^*) + b(y_t - y^{eq}) = 0 \tag{ZBL}$$

which is decreasing with slope $-b/(1+a)$. Above this line (MP) gives $i_t = r^{eq}(\mu) + \eta + \pi_t + a(\pi_t - \pi^*) + b(y_t - y^{eq}) > 0$, below the line it gives $i_t = 0$. Inserting (MP) in the range $i_t > 0$

into (IS) we get the decreasing portion of AD:

$$\begin{aligned} y_t &= \mu - \beta[r^{eq}(\mu) + \eta + a(\pi_t - \pi^*) + b(y_t - y^{eq})] \\ &= y^{eq} - \beta(\eta + a(\pi_t - \pi^*) + b(y_t - y^{eq})) \\ y_t - y^{eq} &= -\frac{a\beta}{1+b\beta}[\pi_t - (\pi^* - \frac{\eta}{a})] \end{aligned}$$

which goes through the point $(y, \pi) = (y^{eq}, \pi^* - \eta/a)$. In particular for $\eta = 0$ it passes through (y^{eq}, π^*) independently of μ. We also observe that it is steeper than the *ZLB* line.[2] Since below that line we have $r_t = 0 - \pi_t$, by inserting (MP) into (IS) we get the following kinked AD curve in the (y, π) plane:

$$y_t = \begin{cases} y^{eq} - \frac{a\beta}{1+b\beta}[\pi_t - (\pi^* - \frac{\eta}{a})] & \textit{above ZLB} \\ \mu + \beta\pi_t & \textit{below ZLB} \end{cases} \tag{AD}$$

It is piecewise linear, continuous by construction, decreasing above the *ZLB* line and increasing below.

The equilibrium of the economy at t is essentially an *AS-AD* equilibrium: it is the intersection of (PC) and (AD). We say that the equilibrium is *stable* if inflation expectations are realized. Recall that expectations are realized only at $y_t = y^{eq}$. The idea of stability is that if expectations are not realized then $\pi^e_{t+1} = \pi_t \neq \pi^e_t$ so that the PC curve - through (y_t, π^e_t) - shifts and the equilibrium changes.

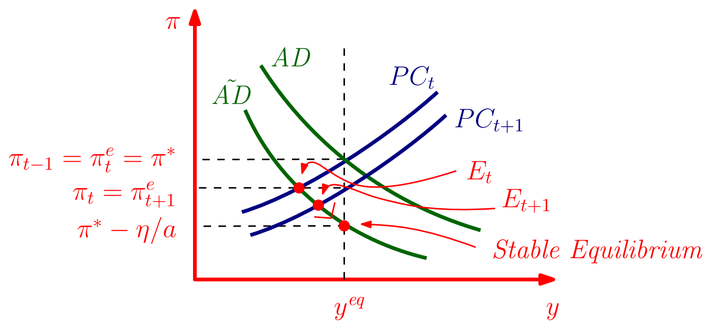
3. Adverse demand shocks. Such shocks are represented in this model by a fall in the μ parameter, which becomes $\mu' < \mu$. A demand shock necessarily affects the bank's estimation error. To see how suppose at the initial μ we have $\eta = 0$; after the shock the equilibrium real rate becomes $r^{eq}(\mu') < r^{eq}(\mu)$ so the current estimate $\hat{r}^{eq} = r^{eq}(\mu)$ correct at μ becomes biased upward as an estimate of $r^{eq}(\mu')$: the error jumps to $\eta = r^{eq}(\mu) - r^{eq}(\mu') > 0$. Therefore the decreasing portion of the AD curve shifts down, since it goes through $(y^{eq}, \pi^* - \eta/a)$. The idea is that the AD curve shifts down because the bank's target rate becomes too high in the worsened demand conditions.

Simplified model. To understand the basic working of the model let us ignore the zero lower bound for now, so that only the decreasing portion of the AD is relevant. Suppose first the bank does not change its estimate of the equilibrium rate so that $\eta > 0$ remains constant. The situation is illustrated in Figure 1.

Starting from a stable equilibrium (y^{eq}, π^*) with $\eta = 0$ the shock at t shifts the AD curve to $\tilde{AD}$, and equilibrium at t is at the intersection between this and the PC_t curve, denoted by E_t in the figure; then the PC curve shifts down to PC_{t+1} since realized inflation is lower than

[2] We want $\frac{1+b\beta}{a\beta} > \frac{b}{1+a}$ which is $(1+b\beta)(1+a) > ba\beta$ which is true.

Figure 1: No ZLB, no learning

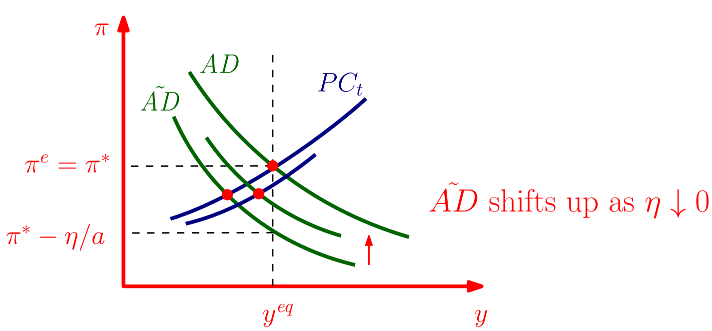


expected, and equilibrium at $t+1$ is at the point E_{t+1}; the process continues and the economy converges to the stable equilibrium at $(y^{eq}, \pi^* - \eta/a)$ where as we know $\eta = r^{eq}(\mu) - r^{eq}(\mu')$. The fact that the bank does not update its estimate is however implausible; indeed the fact that the AD curve goes through $(y^{eq}, \pi^* - \eta/a)$ immediately implies that *in a stable equilibrium the constant* π *satisfies*

$$\pi \lesseqqgtr \pi^* \iff \eta \gtreqqless 0.$$

Therefore the bank can learn the sign of the estimation error from that of $\pi - \pi^*$ at $y = y^{eq}$. So what happens after the shock? The bank cannot immediately realize its estimate has become incorrect, but since the economy enters a path (along $\tilde{AD}$) where $y < y^{eq}$ grows and $\pi < \pi^*$ falls, the bank will deduce that it is moving towards a new stable y^{eq} with a lower constant inflation $\pi < \pi^*$ and therefore learns that necessarily $\eta > 0$ although it does not know the true value of μ; so it starts lowering its estimate $\hat{r}^{eq}$ to correct it. This means of course reducing η that is shifting the AD curve upwards. The learning-and-stimulating process continues until the economy reaches again an equilibrium where $y = y^{eq}$ and $\pi = \pi^*$, *with a lower real rate* $r^{eq}(\mu')$ correctly estimated (since at that point $\eta = 0$ again). The new error-free equilibrium is thus reached through learning. See Figure 2. Of course if the μ parameter goes back up the process will start again in the opposite direction.

Figure 2: No *ZLB* and learning



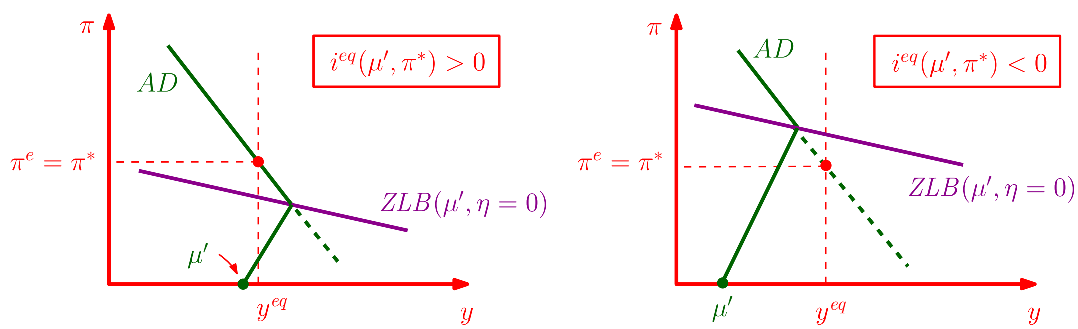
The complete picture. To simplify exposition we will assume that the bank learns the new equilibrium value of r after one period. In other words after the shock at t we have $\eta_t = r^{eq}(\mu) - r^{eq}(\mu')$ and the E_t equilibrium is as before at the intersection of PC_t and $\tilde{AD}$ but then $\eta_{t+1} = 0$ so at that point the decreasing portion of AD passes again through (y^{eq}, π^*).

The formal analysis of the model involves looking at a bunch of straight lines and their relative positions. The bottom line is that whether the central bank can or cannot restore equilibrium depends on how much the shock lowers the equilibrium nominal interest rate:

Bottom Line. *There are two thresholds $0 < i_L < i_H < i^{eq}(\mu, \pi^*)$. If $i^{eq}(\mu', \pi^*) > i_H$ then the central bank can restore the (y^{eq}, π^*) stable equilibrium after correcting its estimation error. If $i_L < i^{eq}(\mu', \pi^*) < i_H$ then the above stable equilibrium may or may not be restored, depending on the slope of the PC curve; the flatter this is the more likely is the bank to succeed. If $i^{eq}(\mu', \pi^*) < i_L$ then output will necessarily remain below y^{eq} and the economy will enter a spiral of declining inflation and output. In such a scenario the only options to restore equilibrium with $y = y^{eq}$ are either raising the inflation target to a $\tilde{\pi}^*$ such that $i^{eq}(\mu', \tilde{\pi}^*) \geq i_L$ or resorting to a fiscal stimulus which raises μ' to a higher $\tilde{\mu}'$ which again makes $i^{eq}(\tilde{\mu}', \pi^*) \geq i_L$.*

We turn to the geometry of the model. Consider the after-shock situation at $\eta = 0$. We know that the error-free ZLB line can be written as $i^{eq}(\mu', \pi^*) + (a+1)(\pi_t - \pi^*) + b(y_t - y^{eq}) = 0$; thus the point (y^{eq}, π^*) is above the line if $i^{eq}(\mu', \pi^*) > 0$ and below the line if $i^{eq}(\mu', \pi^*) < 0$. On the other hand the decreasing portion of the error-free AD curve (steeper than the ZLB line) passes through that point. Therefore the intersection of the two curves is on the right of y^{eq} if $i^{eq}(\mu', \pi^*) > 0$, on the left of it if $i^{eq}(\mu', \pi^*) < 0$. The part of the AD below the line is its increasing portion $\mu' + \beta\pi_t$. Figure 3 illustrates.

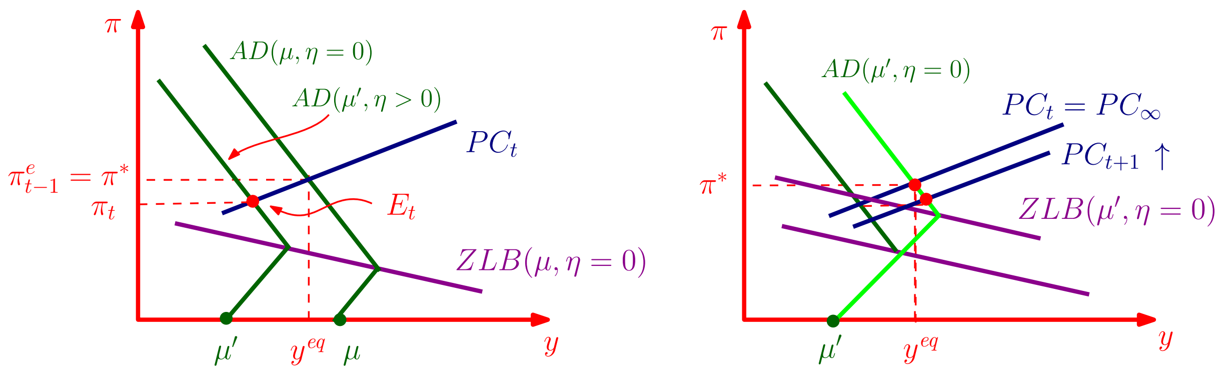
Figure 3: The situation after the shock and after the error is corrected ($\eta = 0$)



The period when the shock occurs the bank has not revised its estimate $\hat{r}^{eq} = r^{eq}(\mu)$ so that the error $\eta = r^{eq}(\mu) - r^{eq}(\mu') > 0$ is generated. The AD shifts down and the ZLB line remains the original one $r^{eq}(\mu) + \pi_t + a(\pi_t - \pi^*) + b(y_t - y^{eq}) = 0$ (where now $r^{eq}(\mu) = r^{eq}(\mu') + \eta$).

In the subsequent period the bank corrects the error and AD and the ZLB line become those we have just seen. The situation in the shock period is in the left panel of Figure 4.[3] The fact that we start from a stable error-free equilibrium implies that $i^{eq}(\mu, \pi^*) > 0$ because otherwise the AD would bend backwards on the left of y^{eq}.

Figure 4: The situation with $i^{eq}(\mu', \pi^*) > 0$



After the shock the AD shifts down to $AD(\mu', \eta > 0)$ - where the arguments indicate that now $\eta = r^{eq}(\mu) - r^{eq}(\mu')$. The economy moves to the equilibrium E_t as before. At this point the bank corrects the error and the new AD and ZLB become the ones at μ' with $\eta = 0$, the decreasing portion of AD again passing through (y^{eq}, π^*). We first consider two "typical" cases, one with $i^{eq}(\mu', \pi^*) > 0$ where the bank restores equilibrium and the other with $i^{eq}(\mu', \pi^*) < 0$ where the economy enters the falling spiral; the existence of the stated thresholds will then be easily understood.

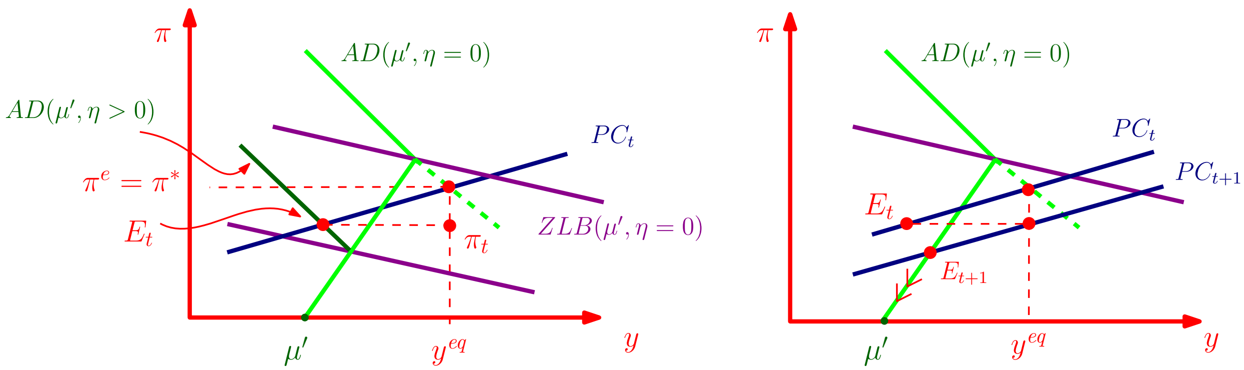
In the right panel of Figure 4 the new ZLB is drawn below the (y^{eq}, π^*) point - which as we know means $i^{eq}(\mu', \pi^*) > 0$ - and the stable equilibrium is restored (with a lower real rate of course): at $t+1$ the PC curve PC_{t+1} passes through $(y^{eq}, \pi^e_{t+1}) = (y^{eq}, \pi_t)$; the intersection with the AD curve, which is then $AD(\mu', \eta = 0)$, is with $y > y^{eq}$ and $\pi_{t+1} > \pi_t$; so the PC curve moves up and at convergence - denoted PC_∞ - it is the same as the original one, and the equilibrium is (y^{eq}, π^*).

Suppose on the contrary that the shock is so severe that $i^{eq}(\mu', \pi^*) < 0$; the new ZLB line is above the (y^{eq}, π^*) point as in the right panel of Figure 3. In this case initially the situation is again as in the left panel of Figure 4, and the economy moves to E_t. But now after the bank corrects the error the situation becomes as drawn in Figure 5.

The AD curve is now $AD(\mu', \eta = 0)$, and E_t is well below the zero lower bound so policy prescribes setting $i_t = 0$. By so doing the bank does stimulate the economy a little, pushing it to E_{t+1} at the intersection of $AD(\mu', \eta = 0)$ and the PC_{t+1} curve, see the right panel of the figure. But then the downwards spiral begins: the PC curve shifts down with expectations

[3]Notice that the PC curve is flatter than the increasing portion of AD by the assumption $\delta < 1/\beta$.

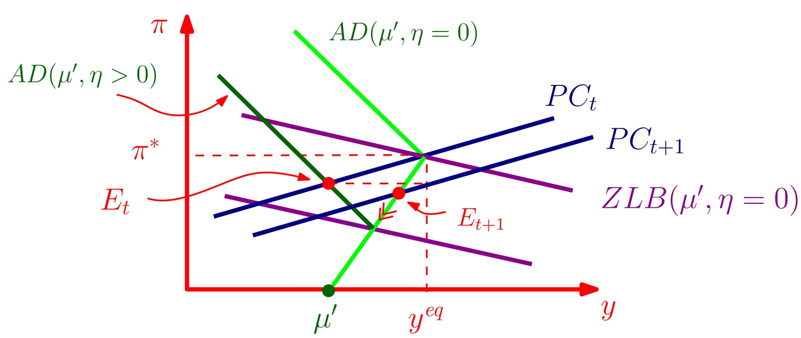
Figure 5: The situation with $i^{eq}(\mu', \pi^*) < 0$



so next period equilibrium will be further down along the increasing portion of the new AD curve; and the downwards spiral does not stop. That the options to restore equilibrium at $y = y^{eq}$ are the ones we have stated - raising inflation target or resorting to fiscal policy - is now clear: what is needed is to somehow bring back the new equilibrium nominal rate into the positive range.

We have seen that if $i^{eq}(\mu', \pi^*) < 0$ the economy necessarily enters the falling spiral. Consider now the case where $i^{eq}(\mu', \pi^*) = 0$ exactly, depicted in Figure 6. The new ZLB line passes through (y^{eq}, π^*), and the PC_{t+1} curve's height at y^{eq} is that of E_t; therefore its intersection with the relevant curve $AD(\mu', \eta = 0)$ is *to the left of* y^{eq}, thus with $\pi_{t+1} < \pi_t$ - and the falling spiral is triggered. The situation is the same for $i^{eq}(\mu', \pi^*) > 0$ small enough (for the gap between π^* and the height of E_t is fixed), whence the threshold i_L.

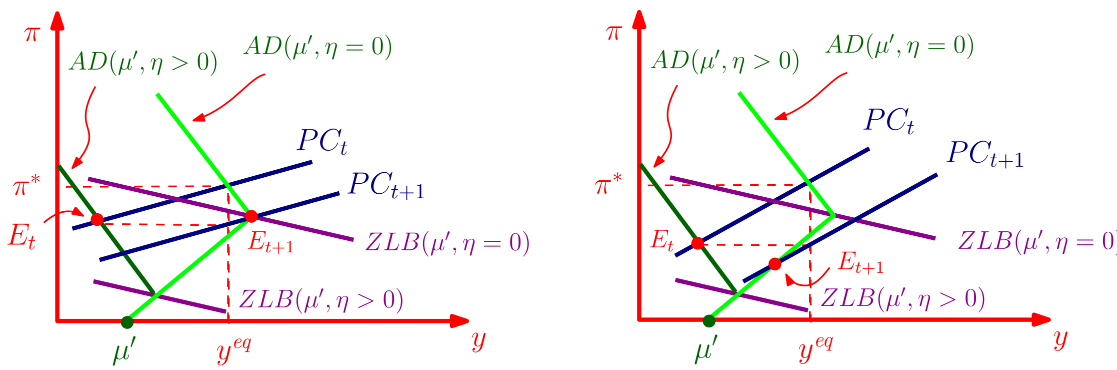
Figure 6: The situation with $i^{eq}(\mu', \pi^*) = 0$



The existence of the i_H threshold follows similarly: from the left panel of Figure 4 we see that if the shock is small enough the new ZLB line is close to the original one so the point (y^{eq}, π_t) is above it, hence the PC_{t+1} curve intersects the new AD to the right of y^{eq}; thus $\pi_{t+1} > \pi_t$, the PC curve moves up and the economy converges to (y^{eq}, π^*).

Consider lastly the middle range $i_L < i^{eq}(\mu', \pi^*) < i_H$. The two possibilities - success and failure - are depicted in Figure 7. In the two panels everything is the same except the slope of the PC curve. In the left panel the curve is flat enough that π_t (the height of E_t) is higher than the value of the new AD at y^{eq}; therefore the PC_{t+1} curve intersects AD on the right of y^{eq} (it does not matter if in the increasing or decreasing part) and the economy converges to (y^{eq}, π^*). In the right panel the PC curve is steeper and the height of E_t is below the value of AD at y^{eq}, so that the PC_{t+1} curve meets AD on the left of y^{eq}, inflation $\pi_{t+1} < \pi_t$ and the economy enters the downwards spiral.

Figure 7: The situation in the range $i_L < i^{eq}(\mu', \pi^*) < i_H$



Comment. It is not our intent to discuss policy at any length. Only one comment on the problem of raising the inflation target: with adaptive expectations as we have defined them it is impossible to change agents' expectations by simply declaring a different target. A possible starting point is to modify expectation formation by assuming that

$$\pi_t^e = (1-\gamma)\pi_{t-1} + \gamma\pi^*$$

where the γ coefficient reflects the credibility of the bank in pursuing the target.

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