Cooperation in oligopoly

In the constituent game there are two firms $i = 1, 2$ which choose to produce quantity $q_i \geq 0$, for simplicity at zero cost, and that market price is $p(q_1, q_2) = 1 - q_1 - q_2$. Thus firm i payoff (its profit) is $\pi_i(q_1, q_2) = p(q_1, q_2)q_i$. The best responses $b_i(q_j)$ are given by the solutions of the maximization problems; setting derivative of π_i equal to zero gives $0 = 1 - q_j - 2q_i$ so that $b_i(q_j) = (1 - q_j)/2$, whence the Nash equilibrium is found as usual as the solution of the system $q_i = (1 - q_j)/2, i \neq j = 1, 2$ which gives $q_i^* = 1/3, i = 1, 2$. In this equilibrium both firms have payoff $\pi_i(q_1^*, q_2^*) = 1/9$.

(a) Compute what the two firms can achieve by colluding to share the maximized joint profits $\pi_1 + \pi_2 = [1 - (q_1 + q_2)](q_1 + q_2) = (1 - q)q$ with $q = q_1 + q_2$. (The answer is that they would produce $1/4 < 1/3$ and get $1/8 > 1/9$ each.

(b) Can they achieve this outcome in a repeated game setting? The natural way to proceed is to try with trigger strategies, and to do this we can model the repeated interaction by setting up a 2×2 constituent game where the firms can either cooperate, action C, by choosing $\hat{q}_i$, or defect, action D, by choosing $b_i(q_j)$. We know the payoffs from CC and DD profiles (both defecting amounts to playing Nash); compute what each gets in DC. (the D-firm produces $3/8 > 1/3$ and gains $9/64 > 1/8$. The C-firm gets $3/32 < 1/9$.

(c) Multiply all payoffs by $64 * 9 = 576$, write down the resulting Prisoners Dilemma and show that cooperation is sustainable in this case if $\delta \geq 9/17$.

Comments on the economics of the game.

1. Notice price and quantities. In the non-cooperative Nash equilibrium total quantity produced is $q_1^* + q_2^* = 2/3$ and the corresponding price is $p(q_1^*, q_2^*) = 1/3$. In the cooperative sustainable outcome quantity is $\hat{q}_1 + \hat{q}_2 = 1/2 < q_1^* + q_2^* = 2/3$ and $p(\hat{q}_1, \hat{q}_2) = 1/2 > p(q_1^*, q_2^*) = 1/3$. So if firms profitably collude they do so at consumers' expenses. This is the basis of anti-trust laws, which aim to prohibit such collusive behaviour. As you can imagine this is not easy because firms do not need an explicit contract to implement this kind of trigger strategies, and the authorities have usually to break collusion by trying to prove that there is an *implicit* contract among the offending firms. This is all the more true when the firms are not operating within national borders of course.

2. These days (end 2016), as in several other moments, oil producing countries are under the spotlight: they are negotiating to curb oil production. One thing to notice is that these agreements, besides being much more complex than in our simple model, do break apart. Why is this, given that they can be sustained in a *subgame perfect* equilibrium? The answer is that the degree of patience of the players changes - remember that for our result we need enough of it. When a player is in dire need of cash his patience may decrease to the extent that current gains from deviation become larger than future losses from retaliation, and in that case equilibrium breaks down as predicted by the model.