1. To show that $(\hat{a}_1, \hat{a}_2)$ is Nash.

For any a_2 1's best response $BR_1(a_2) \leq \hat{a}_1$ - this is clear. And for any $a_1 < \hat{a}_1$ we have

$$u_1(a_1, \hat{a}_2) = -a_1 - L(a_1, \hat{a}_2) \leq -\hat{a}_1 - L(\hat{a}_1, \hat{a}_2) < -\hat{a}_1 = u_1(\hat{a}_1, \hat{a}_2)$$

the first inequality because $(\hat{a}_1, \hat{a}_2)$ is maximum. So $BR_1(\hat{a}_2) = \hat{a}_1$. As to player 2 we have

$$u_2(\hat{a}_1, a_2) = -a_2 - L(\hat{a}_1, a_2) \leq -\hat{a}_2 - L(\hat{a}_1, \hat{a}_2) = u(\hat{a}_1, \hat{a}_2)$$

again by definition of max. Hence $BR_2(\hat{a}_1) = \hat{a}_2$.

2. To show that $(\hat{a}_1, \hat{a}_2)$ is the unique Nash.

We know that $BR_1(a_2) \leq \hat{a}_1$ for any a_2; we now show that for any $a_1 \leq \hat{a}_1$ we have $BR_2(a_1) = \hat{a}_2$. Hence in any Nash $a_2 = \hat{a}_2$, and since $BR_1(\hat{a}_2) = \hat{a}_1$ we get uniqueness. Lastly: $BR_2(\hat{a}_1) = \hat{a}_2$ we know. For $a_1 < \hat{a}_1$: for $a_2 < \hat{a}_2$ we have

$$u_2(a_1, a_2) = -a_2 - L(a_1, a_2) < -a_2 - L(\hat{a}_1, a_2) \leq -\hat{a}_2 - L(\hat{a}_1, \hat{a}_2) < -\hat{a}_2 = u_2(a_1, \hat{a}_2)$$

the first inequality because L is decreasing and the second by definition of maximum; for $a_2 > \hat{a}_2$ clearly $u_2(a_1, a_2) = -a_2 < -\hat{a}_2 = u_2(a_1, \hat{a}_2)$; hence for $a_1 < \hat{a}_1$ too $BR_2(a_1) = \hat{a}_2$. That's it.