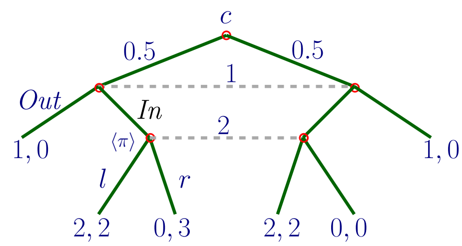
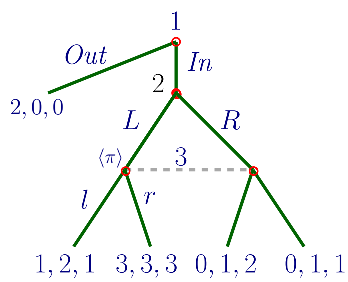
Example 1

Consider the game depicted in the figure below. (a) Is (Out, r) a Nash equilibrium of the game? (b) Is there a π which makes it Perfect Bayesian? (c) Is there a π which makes it Sequential?



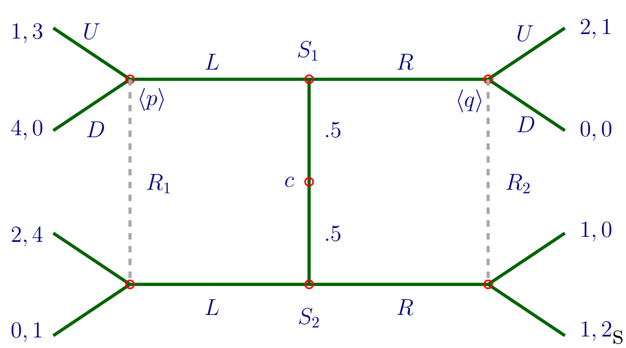
Example 2 (Abby Friedman?)

Consider the game in the figure below. (a) Write the normal form of the game between 2 and 3 starting after 1 plays In and check that the only Nash equilibrium is L, r. Therefore the only subgame perfect equilibrium of the whole game is In, L, r. (b) Is the strategy profile Out, L, l Nash? (c) Is the last profile part of a PBE? (d) Is it part of a sequential equilibrium? (e) Can 2 play R in a PBE?



Example 3 (Abby Friedman?)

Consider the game in the figure below. It may be interpreted as a sender-receiver game (a so-called signaling game) where the sender receives a private information about herself (her "type") and sends a signal to the receiver who observes the signal but not the sender's type. For simplicity the initial nature move is chosen with 50-50 probability. The first payoff is the sender's. Receiver's beliefs at her two information sets are denoted by p and q.



We look for pure strategy PBE's. Strategies are written in the same order as the information sets, so for example LR means 1 plays L at her upper node and

R at her lower one. You are required to determine PBE's, if any, where 1 plays LL, RR, LR, RL. So start with LL and see if this is part of an equilibrium, etcetera.

Example 4 (Zack Grossman?)

The structure of the game is the same as the previous one. S can again be of two types: now S_1 is a strong guy, S_2 is not. He challenges R to fight, and R can pass P or accept A. If R passes he gets zero, if he accepts he gets -2 with S_1 and 1 with S_2. S gets 1 if R passes independently of his type, while if R meets S_1 gets 0 but S_2 gets -1. S can signal strength when challenging, l, or not, r; and S_2, the weak type, has cost γ to send the strength signal (so if S_2 chooses l he gets $1-\gamma$ if R passes and $-1-\gamma$ if R meets). Again we look for pure strategy equilibria.

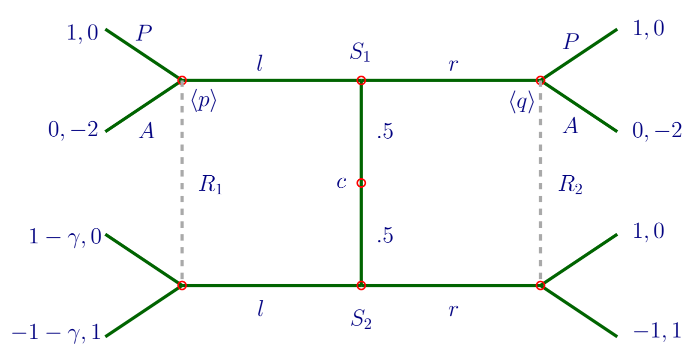
Draw the game, and: (a) Find the values of γ for which there is a PBE where S plays ll; (b) Find the values of γ for which there is a PBE where S plays lr. (c) Notice that S_1 is indifferent between l and r; is there an equilibrium where he plays r? (d) Is ll part of a sequential equilibrium?

Solution to Example 1. (a) Yes: given r player 1 strictly prefers *Out* to *In*; and given *Out* player 2 is indifferent between l and r. (b) Yes, any π such that $2 \leq 3\pi$ that is $\pi \geq 2/3$. (c) No. For any strategy where *In* is played with positive probability the implied $\pi = 1/2$ so any consistent belief should have $\pi = 1/2$, and this makes 2 prefer l to r.

Solution to Example 2. (a) Just check. (b) Yes: given the strategies of 2 and 3 player 1 strictly prefers *Out* to *In*, and 1 and 2 are indifferent among all their strategies given *Out*. (c) Yes. For 3 to prefer l we need $\pi + 2(1-\pi) \geq 3\pi + 1 - \pi$ that is $\pi \leq 1/3$; and 2 prefers L to R for L is dominant for her. Therefore PBE's need not be subgame perfect. The problem is that they do not constrain beliefs out of equilibrium, and in particular beliefs can be inconsistent with 2 playing her dominant strategy. (d) No, because if L is played with a probability which tends to 1 then the implied $\pi \to 1$ too in which case 3 prefers r to l. (e) No, by sequential rationality.

Solution to Example 3. Observe first that sequential rationality implies that in any equilibrium R_1 plays U. [1] Given this, in any equilibrium S_2 must play L. For S_1 to play L it must be the case that R_2 plays D, which she can if $q \leq 2/3$. So a PBE has S playing LL and R playing UD with $q \leq 2/3$ (and $p = 1/2$). We have excluded already RR and LR (since S_2 must play L so we are left to check RL. In this case R knows where she is; so R_1 must play U (as always) and R_2 must play U as well. Given R's play S strategy is indeed optimal so RL, UU is the other PBE (with $p = 0, q = 1$).

Solution to Example 4 (left to you to work this out fully). (a) $\gamma \leq 2$; (b) $\gamma \geq 2$; (c) Yes, he can play rr; (d) Yes. The game is the following:



[1]Here "R_1 plays U" means R plays U at her left information set. Analogous terminology used in the sequel.