Decision Theory, S Modica February 2011
Principal-Agent: Insurance and Moral Hazard

We have seen (P. Wakker's Prospect Theory p.73) that a risk neutral insurer and a risk averse agent who faces a risk of accident may write a mutually beneficial insurance contract, because to get rid of the risk the agent is willing to pay more than its expected value, which in turn is the insurer's expected cost.

More precisely, if an accident with damage $d > 0$ occurs with probability p the agent is willing to pay up to $pd + \pi$, where pd is expected value of damage and π is the risk premium associated with the prospect, whereas the least the insurer would be willing to accept is dp.

Therefore they can write a contract (q, R) whereby the agent pays $q \in (pd, pd+\pi)$ to the insurance and the latter pays $R = d$ to the agent *in case the accident occurs*. Under this contract the agent is fully insured, for she will get $\bar{w} - q$ in any case, $\bar{w}$ being her initial income; the insurer gets q if the accident does not occur and $q - R$ if it occurs.

Thus the insurer's payoff under the contract is the expected profit $q - pR$ which is positive: $q - pR = q - pd > 0$, so the contract is beneficial to the insurer. As to the agent, with no insurance she would get expected utility $(1 - p)u(\bar{w}) + pu(\bar{w} - d)$, where u is her vNM utility and $\bar{w}$ her initial income; while under the contract her payoff is

$$u(\bar{w} - q) > u(\bar{w} - pd - \pi) = (1 - p)u(\bar{w}) + pu(\bar{w} - d)$$

(inequality from $q < pd+\pi$), so the contract is beneficial to the agent as well. The insurance contract implements a Pareto improvement.

Of course this is not the end of the story *if the agent can influence probability of accident by paying more or less attention*. For then with no insurance she will exert due care, while if insured she might not. So if the insurer observes no-insurance probability of accident and writes a contract based on that value she will possibly go broke, because the realized probability of accident will likely be higher than estimated value. On the other hand, realizing this fact, the insurer/principal will try to design a contract (q, R) taking into account the agent's incentives, and the fact that she must be willing to sign the contract. This is the archetype of the moral hazard problem, which we shall analyze in some detail in this exercise. In particular, two questions are relevant: (a) Does a non-null contract $(q, R) \neq (0, 0)$ exist under these constraints? And (b) Will the principal induce more or less attention on the part of the agent than she would exert without insurance?

The answer to the first question is not obvious, because in principle it may be the case that the unobserved effort the principal can optimally induce is so low, hence the probability of accident so high, that no non-null contract exists giving the insurer a positive profit.

To state the problem let $a \geq 0$ be the effort/attention level, $p(a)$ the resulting probability of accident, and $\psi(a)$ the (utility) cost of effort. On the involved functions u, p, ψ we make the following

Assumption. *All functions are smooth. The vNM utility u is increasing concave. Effort cost ψ is increasing convex, with $\psi(0) = \psi'(0) = 0$. Accident probability p is decreasing convex, with $p(0) = 1$ and $p'(0) < 0$.*

As usual it will help to draw these functions. Notice that effort level, unlike the cases we have seen in class, is not binary; this will allow us to use calculus, and does not change the nature of the incentive and participation constraints.

We start with the no-insurance situation where the agent is on her own, because the utility level she will reach in this case will be the u_0 that the insurer will have to use as the agent's reservation utility in the principal-agent problem she will have to solve. In this case the agent's expected utility is

$$(1-p(a))u(\bar{w})+p(a)u(\bar{w}-d)-\psi(a)=u(\bar{w})-\big(p(a)\Delta u_0+\psi(a)\big)\,,$$

with $\Delta u_0 = u(\bar{w}) - u(\bar{w}-d)$. Thus the agent's problem, to maximize expected utility, may be written as

$$\min_a\ p(a)\Delta u_0+\psi(a)\,.$$

Question 1. *Show that the first order condition characterizes the global optimum, and that it implies that optimal a, say a_0, is positive and increasing in Δu_0. Letting $u_0 = u(\bar{w}) - \big(p(a_0)\Delta u_0 + \psi(a_0)\big)$ be the reached level of utility, show that $u_0 > u(\bar{w}-d)$. Finally, show that $0 < p(a_0) < 1$.*

Solution. The objective function is convex, so the first order condition characterizes global optimum. It is given by $\psi'(a)+\Delta u_0\, p'(a) \geq 0$, with equality if $a > 0$. But at $a = 0$ we have $\psi'(0)+\Delta u_0\, p'(0) < 0$, so optimum is with $a > 0$ and is characterized by the FOC

$$-\frac{\psi'(a)}{p'(a)}=\Delta u_0\,.$$

That the a satisfying this condition is increasing in Δu_0 is direct from convexity of ψ and p. For the last point, by convexity all $a \neq a_0$ give lower payoff, so in particular $u_0 > u(\bar{w}) - \big(p(0)\Delta u_0 + \psi(0)\big) = u(\bar{w}-d)$. Lastly, $p(a_0) < 1$ from $a_0 > 0$; if $p(a) = 0$ then $p'(a) = 0$ so $\psi'(a) + \Delta u_0\, p'(a) > 0$ whence a cannot be optimal. □

Now consider the insurance problem. The principal will maximize expected profit under agent's incentive and participation constraints. The incentive constraint is that the agent must choose the action that maximizes his utility under the contract (q, R) offered by the insurer. The agent's expected utility is now

$$(1-p(a))u(\bar{w}-q)+p(a)u(\bar{w}-q-(d-R))-\psi(a)=u(\bar{w}-q)-\big(p(a)\Delta u+\psi(a)\big)\,,$$

where now $\Delta u = \Delta u(q,R) = u(\bar{w}-q) - u(\bar{w}-q-(d-R))$. Note that no-insurance is a special case, with $\Delta u_0 = \Delta u(0,0)$ and $a_0 = a(0,0)$. Action is chosen to solve

$$\min_a\ p(a)\Delta u+\psi(a)\,,$$

where notice that optimal action depends on (q,R) only through Δu: $a(q,R) = a(\Delta u(q,R))$.

Question 2. *Show that $a(q,R) > 0$ if and only if $\Delta u > 0$, i.e. iff $R < d$ –less than full insurance. Show that in this case the optimum is again characterized by FOC, which now implies that $a(q,R)$, unique, is increasing in q and decreasing in R.*

Solution. If $\Delta u \leq 0$ it is $\psi'(a) + p'(a)\Delta u > 0$ for all $a > 0$, so $a = 0$ at the optimum. Taking now $\Delta u > 0$, local optimum is global again by convexity (given $\Delta u > 0$). It is easy to establish (using monotonicity and convexity of u) that $\Delta u(q,R)$ is increasing in q and decreasing in R, from which the result follows. Uniqueness follows by convexity. □

The participation constraint is as usual. So the principal's problem is

$$\max_{(q,R)}\ q-p(a)R$$

subject to

$$a=\arg\max_a\ u(\bar{w}-q)-\big(p(a)\Delta u(q,R)+\psi(a)\big)$$
$$u(\bar{w}-q)-\big(p(a)\Delta u(q,R)+\psi(a)\big)\geq u_0\,.$$

Question 3. *Show that in the set of contracts (q,R) with $R \geq d$ the optimal one is $(0,0)$.*

Solution. We have seen that with $R \geq d$ the agent chooses $a = 0$, which implies $p(a) = 1$. Then the principal's payoff is $q - R$, which is non-negative iff $q \geq R$. But then the agent's payoff is $u(\bar{w} - q - d + R) \leq u(\bar{w} - d) < u_0$. □

This shows that the principal can, and will, restrict attention to partial-insurance contracts with $R < d$, and so shall we. Note that in this class $\Delta u(q, R) > 0$ so optimal action is as in Question 2. At this point you are hopefully well into the problem, and can answer our original questions (a) and (b).

Question 4. *Show that the principal will implement a non-null contract.*

Hint: Show that the principal can make the participation constraint slack without making losses.

Solution. It suffices to show that the principal can do better than offering the zero contract. Starting from this zero-profit contract consider raising R and q so as to maintain profit at zero, i.e. setting $q(R) = p(a)R$, where $a = a(q, R)$; then $\partial q/\partial R = p$. The result is proved if by so doing the agent's payoff locally increases (from u_0 which she gets at $(0, 0)$), because the principal can do better (at least as well as) than moving away from $(0, 0)$ the way we have assumed she is doing. With the policy under consideration the change in agent's payoff at $(0, 0)$ is

$$\frac{\partial}{\partial R}\big[(1 - p(a))u(\bar{w} - q(R)) + p(a)u(\bar{w} - d - q(R) + R) - \psi(a)\big]\Bigg|_{(0,0)}$$

$$= p(a)(1 - p(a))\big(u'(\bar{w} - d) - u'(\bar{w})\big) + \frac{\partial}{\partial a}[\cdot]\frac{da}{dR}\,;$$

But the last $\frac{\partial}{\partial a}[\cdot] = 0$ from FOC, so the envelope theorem leaves us with the first term, which is positive by concavity of u and $0 < p(a(0, 0)) < 1$. □

This shows that given risk neutrality of the insurer and risk aversion of the agent, Pareto improving insurance is still possible when action is not observed. Last, the level of care: our last exercise will show that the level of care is lower with insurance than without:

Question 5. *Show that for the (q, R) optimally implemented it is $a(q, R) < a_0$.*

Hint: first show that as Δu increases, although a increases (making $p(a)$ decrease), the value $p(a)\Delta u + \psi(a)$ also increases.

Solution. We know that $a(q, R)$ depends only on Δu. Let $\Delta u_1 \leq \Delta u_2$, and $a_1 \equiv a(\Delta u_1) \leq a(\Delta u_2) \equiv a_2$. Then $p(a_2) \leq p(a_1)$, but

$$p(a_2)\Delta u_2 + \psi(a_2) \geq p(a_2)\Delta u_1 + \psi(a_2) \geq p(a_1)\Delta u_1 + \psi(a_1)\,,$$

where the first inequality follows from $p(a_2) \geq 0$ and the second from minimization.

Letting (q, R) be the optimal contract, we proceed by contradiction: if $\Delta u(q, R) \geq \Delta u_0$ then $p(a(q, R))\Delta u + \psi(a(q, R)) \geq p(a_0)\Delta u + \psi(a_0)$; but for any $q > 0$ it is also $u(\bar{w} - q) < u(\bar{w})$, so agent's payoff $u(\bar{w} - q) - \big(p(a(q, R))\Delta u(q, R) + \psi(a(q, R))\big) < u_0$; hence the only (q, R) satisfying the agent's participation constraint and giving the principal non-negative profit would be $(0, 0)$. This contradicts the result in Question 4. Thus $\Delta u(q, R) < \Delta u_0$ whence $a(q, R) < a_0$, as was to be shown. □