The Rock-Paper-Scissors game

The game is in the matrix below, where only the payoff of player 1 is reported since that of player 2 is just the opposite:

	R	***P***	***S***
R	0	-1	1
P	1	0	-1
S	-1	1	0

Denote a mixed strategy of 1 as (p_1, p_2, p_3) and of 2 as (q_1, q_2, q_3). To find the equilibrium:

1. Show there is no pure-strategy equilibrium.
2. Show there is no equilibrium where one of the players mixes over two strategies only.

   To do this suppose 2 does not play for instance S - that is $q_1 + q_2 = 1$ with $q_1, q_2 > 0$. Show that then 1 will not play R - the one which beats S (hint below). But now if 1 does not play R then by the same token 2 should not play P, that is $q_2 = 0$ - contradiction. Since the argument holds for any player and pair of strategies the claim is proved.

   Hint: To show that if 2 does not play S then 1 will not play R find p such that the mixture $(p, P; 1-p, S)$ is strictly better than R for 1 for any q_1. Note that P gives 1 expected payoff of q_1, S gives her $(-1) \cdot q_1 + 1 \cdot q_2$ and R yields $-q_2$; also, use $q_2 = 1 - q_1$. You should get $p > 1/2$.
3. From the previous point it follows that the equilibrium is fully mixed. Now use the indifference conditions to find $p_i = q_i = 1/3$ for $i = 1, 2, 3$.