Three-way duel (Or: the game of your life)

There are three players in a duel: *Jane*, a young girl, who kills with probability 0.3 if she shoots (that's you); *Cowboy* who kills with probability 0.8; and *Küller*, who kills with probability 1. There are two rounds, in each of which each alive player has a shot; first Jane, then Cowboy then Killer. At any round the player who plays may choose to shoot at any alive player or not to shoot. At the end of the second round the players who are alive share a prize M. Players have equal preferences, represented by a u such that $u(0) < u(M/3) < u(M/2) < u(M)$.

(a) Find the subgame perfect equilibrium of the game. In particular, what is Jane going to do in the first round? (No need to draw the whole game!). (b) Compute, for Jane and Killer, the probability of remaining alive at the end of the game (*Answer:* 41.2% and 14% respectively!) and the probability of getting M (*Answer:* 30% and 14% respectively).

Moral of the Story and a Modification of the game

When you, young and small, start moving in a working environment with lots of bigger guys around, don't rush to raise your voice too much. It may be better to stay covered for a while. Unless... well, unless if you skip your first chance you have no other, as in the following modification if the above game.

The modification is simply that if you choose not to shoot in the first round you will not be given the move in the second. Show that in this case Jane has to shoot in the first round. *Hint.* It suffices to show that shooting for example at Killer is better than no shooting. So compute expected payoff from not shooting (*Sol.* $0.84u(0) + 0.16u(M/2)$) and the probability of dying if you shoot at Killer (*Sol.* 0.6852). Conclude from this that shooting is better.