S Modica 8.IV.2002

Investing with a Broker [1]

The Problem. A risk neutral, subjective expected utility maximizer has money to invest. She has the following alternatives:

- deposit it into a bank account and earn zero for sure
- buy a risky asset, which yields 1 or -1 if it goes well or bad —events resp. h and l (for high and low)
- go and see a broker, pay a commission c, listen to his advice, which can be either "buy" —event β— or "don't buy" —event δ—, and decide whether to follow it or not.

A suitable state space for the problem is $S = \{h\beta, h\delta, l\beta, l\delta\}$; and abusing notation we may let $h = \{h\beta, h\delta\}$, $\beta = \{h\beta, l\beta\}$, et cetera. Suppose the decision maker's subjective probability P on S is determined by the following judgments on the quality of asset and broker:

$$\mathsf{P}(h) = 0.6,\ \mathsf{P}(l) = 0.4; \quad \mathsf{P}(\beta \mid h) = \xi_1,\ \mathsf{P}(\delta \mid l) = \xi_2.$$

Note that ξ_1 (resp. ξ_2) measures the ability of the borker to recognize a good (resp. bad) asset. And **assume** that $\xi_i > 0.5,\ i = 1, 2$. This says that the broker has at least some professional skill (in general it is plausible that $\xi_1 > \xi_2$, since good news circulate more easily that bad ones; but we will not need such an assumption).

Determine the optimal decision for $(\xi_1, \xi_2) \in (0.5, 1]^2$.

Solution (details on request). The optimal decision is to invest without the broker if $3\xi_1 + 2\xi_2 < 3 + 5c$, and go see the broker and follow his advice under reverse inequality; on the segment $3\xi_1 + 2\xi_2 = 3 + 5c$ the acts I and F are indifferent. If you like pictures, you can draw one for e.g. $c = 0.1$.

[1] Based on a problem in D.V. Lindley, *Making Decisions*, Wiley 1985.