

### Osborne-Rubinstein def. 89.1 and Trees

Just in case you need to compare this definition with the more common based on the concept of tree.

**Definition.** Binary relation  $\succ$  is *asymmetric* if  $x \succ y \Rightarrow \neg(y \succ x)$ ; it is a *total order* if  $x \neq y \Rightarrow (x \succ y) \vee (y \succ x)$ . A *least element* for  $\succ$  is an  $x$  such that  $x' \succ x$  for all  $x' \neq x$ . By  $x \prec y$  we mean  $y \succ x$ .

A pair  $(H, \succ)$ , where  $H$  is a set and  $\succ$  is a binary relation on it, is a *tree* if  $\succ$  has a least element  $\emptyset$  and for each  $h \in H$  the set  $B(h) = \{h' : h' \prec h\}$  is totally ordered by  $\succ$ .

The set  $H$  in definition 89.1 becomes a tree  $(H, \succ)$  as follows. For  $h = (a^1, a^2, \dots, a^k) \in H$  and  $1 \leq m \leq k$  define its  $m$ -truncation as  $\tau_m(h) = (a^1, a^2, \dots, a^m)$ . Then define  $\succ$  on  $H$  as follows: (i)  $\emptyset \prec h$  for all  $h \neq \emptyset$ ; (ii) for all  $h, h' \neq \emptyset$ ,  $h' \prec h$  if there is  $m < k$  such that  $h' = \tau_m(h)$ .

That's all. So you can call  $h$  a "node" if you like.