The penalty game

The game is very simple: kicker kicks the ball and keeper keeps the goal. Actions are for both left, center, right (kick and dive). Payoffs are scoring probability for kicker and the complement for goalie, all multiplied by 100 to ease reading. They are assumed to be as in the following table, where the kicker plays rows:

		q		
		L	C	R
p	l	$65, 35$	$95, 5$	$95, 5$
	c	$95, 5$	$0, 100$	$95, 5$
	r	$95, 5$	$95, 5$	$65, 35$

Look at the entries. Under l, L and r, R (dive same side of kick) goalie can has a 35% chance to save; with c, C he saves for sure; under any profile where the two make different choices kicker can still kick out, but with 95% probability he scores.

Now also think about the game: there is an intrinsic symmetry between left and right in the game, which should allow us to simplify computations. Indeed the only numbers we need to compute are probabilities p, q of playing left.

To analyze the game observe first that there are no pure strategy equilibria. To see this fix goalie's action, say L to fix ideas; then kicker best reply is a mixture of c, r, but given this then L is dominated by C or R. Similarly we exclude partially mixed profiles. Thus equilibrium must be fully mixed.

(a) Since kicker mixes he should in particular be indifferent between l and r, and (obviously!) this can only happen if the probabilities that the goalie dives left or right must be... Therefore to compute goalie's mixture we need only impose $l \sim_1 c$. Same story to compute kicker's mixture. Thus equilibrium is... (*Answer* $p = q = 95/220 \approx 0.43$) (b) Compute equilibrium probability of scoring - again the exercise is to exploit the game's symmetry and get a simple formula. Observe that this probability is equal to expected kicker's payoff divided by 100. (*Answer* $\approx .82$)