

**Probabilistic reasoning under coherence:
theory and applications.**

SSD MAT/06, 4 CFU, 24 hours
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Course Outline.

This course aims to give an introduction to the coherence approach to probability. It is divided into two parts: theory and applications. The first part will be taught by Giuseppe Sanfilippo and the second part will be thought by Niki Pfeifer.

Part 1: Theory. We start by a brief critical discussion of the various approaches to probability: classical, frequentistic, axiomatic, and subjective. Then, we introduce de Finetti's coherence principle as a basis of subjective probability. In terms of betting, *coherence* means to avoid bets which allow for sure loss. We will learn how the classical properties of (a finitely additive) probability follow from the coherence principle. We deepen the understanding of several selected aspects of coherence, like geometrical interpretations, penalty criterion, proper scoring rules and probability elicitation (exploiting Bregman divergence), conditional events and probability (including conditioning events with zero probability), logical operations among conditional events and nested conditionals. We will discuss different notions of coherence like generalized coherence or total coherence for imprecise probabilities. We will learn about algorithms for imprecise probability and how to use mathematical software (e.g., R, CkC-package, Matlab) for coherence checking and propagation.

Part 2: Applications. We present recent formal and experimental results on applying the coherence principle to understand human reasoning under uncertainty. We model human inference within coherence-based probability logic, which is characterised by transmitting the uncertainty of the premises to the conclusion and how this approach relates to logical validity and Adams' probabilistic validity. Specifically, we discuss formal and experimental work on nonmonotonic reasoning (retracting conclusions in the light of new evidence), conditionals (if-then statements) and compounded conditionals (e.g., *if H then A and if K then B*), categorical syllogisms (e.g., modus barbara, transitivity), conditional syllogisms (e.g., probabilistic modus ponens), and argumentation (e.g., argument strength, Ellsberg-Paradox). Finally, we also discuss the proposed approach in the light of formal epistemology.

¹Hours 16, Home page <http://www.unipa.it/sanfilippo>

²Hours 8, Home page <https://homepages.uni-regensburg.de/~pfn23853/> Ore 8

Further reading

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- [11] Angelo Gilio and Giuseppe Sanfilippo. Generalized logical operations among conditional events. *Applied Intelligence*, 49(1):79–102, 2019.
- [12] Frank Lad. *Operational subjective statistical methods: A mathematical, philosophical, and historical introduction*. Wiley, New York, 1996.
- [13] Frank Lad, Giuseppe Sanfilippo, and Gianna Agró. Extropy: complementary dual of entropy. *Statistical Science (Journal of the Institute of Mathematical Statistics)*, 30(1):40–58, 2015.
- [14] N. Pfeifer and G. Sanfilippo. Probability propagation in selected Aristotelian syllogisms. In G. Kern-Isberner and Z. Ognajović, editors, *Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU 2019)*, volume 11726 of *Lecture Notes in Computer Science*, pages 419–431. Springer, 2019.
- [15] Niki Pfeifer. The new psychology of reasoning: A mental probability logical perspective. *Thinking & Reasoning*, 19(3–4):329–345, 2013.
- [16] Niki Pfeifer and Gernot D. Kleiter. Coherence and nonmonotonicity in human reasoning. *Synthese*, 146(1-2):93–109, 2005.
- [17] Niki Pfeifer and Hanna Pankka. Modeling the Ellsberg paradox by argument strength. In *Proceedings of the 39th Cog. Sci.*, pages 2888–2893, Austin, TX, 2017. The Cog. Sci. Soc.
- [18] Niki Pfeifer and Giuseppe Sanfilippo. Probabilistic squares and hexagons of opposition under coherence. *International Journal of Approximate Reasoning*, 88:282 – 294, 2017.
- [19] Giuseppe Sanfilippo, Niki Pfeifer, David E. Over, and Angelo Gilio. Probabilistic inferences from conjoined to iterated conditionals. *International Journal of Approximate Reasoning*, 93(Supplement C):103 – 118, 2018.