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LPV model identification for gain scheduling control: An application to rotating stall and surge control problem

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Abstract

We approach the problem of identifying a nonlinear plant by parameterizing its dynamics as a linear parameter varying (LPV) model. The system under consideration is the Moore–Greitzer model which captures surge and stall phenomena in compressors. The control task is formulated as a problem of output regulation at various set points (stable and unstable) of the system under inputs and states constraints. We assume that inputs, outputs and scheduling parameters are measurable. It is worth pointing out that the adopted technique allows for identification of an LPV model's coefficients without the requirements of slow variations amongst set points. An example of combined identification, feedback control design and subsequent validation is presented.

Keywords: Gain scheduling; Identification for control; LPV models; Stall and Surge control

1. Introduction

The surge and stall control problem has been widely studied in the literature since the derivation of the well-known Moore–Greitzer model (Moore & Greitzer, 1986). For further background and information, we refer the reader to the extensive surveys reported in Jager (1995) and the book Ref. Gravdahl and Egeland (1999). The Moore–Greitzer model is a simplified three state lumped model obtained from an approximation of the partial differential equations modeling the flow and pressure variations in a compressor. This model is widely recognized in the literature because it captures the essential features of rotating stall and surge phenomena in such compressors.

Suppressing stall and surge phenomena can be cast as a problem of stabilizing or enlarging the basin of attraction of various set points of the Moore–Greitzer model. Physically, this means that compressors can operate in the regions of high pressure rise or low specific power consumption, increasing efficiency and reducing costs. Moreover, larger transients are permitted, thus enabling quicker reactions to changes of operating conditions. Such problems of regulation around various set points can be nicely addressed using a gain scheduling approach, and more specifically the parametrization of nonlinear systems as linear parameter varying (LPV) models (Shamma & Athans, 1991). LPV models are typically obtained from a detailed analysis of the underlying system equations and knowledge of the various parameters and coefficients in the model. In this paper we follow a different approach where identification methods are used to determine the

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model at various set points. In particular we use the identification method, developed in Bamieh and Giarré (2002) for LPV systems, which allows the system to vary rapidly between set points. This algorithm is used to identify the Moore–Greitzer model (in LPV form), and the resulting model is then used for feedback control design.

Our presentation is organized as follows. We first provide a brief overview of the LPV systems and their use for gain scheduling control, with particular attention to LPV identification approaches and gain scheduling predictive control for LPV models. Section two contains a brief description of the stall and surge control problem for compressors. In the third section, the LPV identification problem and the scheduling predictive control are posed and the model derived. In the last section, an example of identification of the Moore–Greitzer model is given with subsequent feedback control design and validation.

1.1. An overview on LPV systems for gain scheduling

In many practical situations, a control system is subject to transitions among different conditions associated to variations of some fundamental parameters: these may account for physical modifications of the plant itself and/or model switchings due to linearization at different operating points. One approach to deal with this is the *gain scheduling*, when the compensator parameters are chosen on-line as function of some exogenous or endogenous signal. Classical control design based on the linearized dynamics need not exhibit good performance or even stabilizing when operating away from the equilibrium. One possibility is to linearize along a trajectory which is not restricted to a local operating region. However, this trajectory must be known in advance in order to perform the control design, and such advance knowledge is often not available. *Gain Scheduling* is then a two-step procedure where: (1) one designs local linear controllers based on linearization of the nonlinear system at several different equilibria (operating conditions) (2) a nonlinear controller for the nonlinear system is obtained by *interpolating* or *scheduling* among the local operations point design.

A useful theoretical framework to study gain scheduling control is the one of LPV systems, a terminology which was first introduced in Shamma and Athans (1991).

A discrete LPV systems is represented in state space as

$$x(k+1) = A(p(k))x(k) + B(p(k))u(k),$$

$$y(k) = C(p(k))x(k) + D(p(k))u(k). \quad (1)$$

The exogenous parameter $p(k)$ is assumed a priori unknown. However, it can be measured or estimated upon operation of the system, see Shamma and Xiong (1999), Shamma (1996), and the recent survey (Rugh & Shamma, 2000), and references therein.

Suppose that some a priori knowledge is available, for instance the range of variation on the exogenous parameter p. A possible control design is to consider the variation of p as uncertainty and to design a single robust controller for the family of systems. This is very conservative, especially if potential variations of p are large. Typically, in order to reduce conservatism, it is convenient to measure the parameter value during operation: design a robust controller around each operating point and switch between controllers according to some gain-scheduling policy. This is a good compromise between performance and robustness and the stability question in the switching zone has been solved in Shamma and Xiong (1999).

An alternative design technique for gain scheduling is the parameter dependent H_∞ controller, based upon the introduction of a LFT representation of the controllers dependent on the parameter $p(t)$. This was introduced and solved in Packard (1994), Apkarian and Gahinet (1995), and Apkarian, Gahinet, and Becker (1995).

In Blanchini (1999), a comparison between gain scheduling and the robust state feedback stabilization problem was studied. Trivial examples exist of LPV discrete-time systems that can be stabilized via gain-scheduling controller but not by means of state feedback controllers which ignore the parameter. These facts motivated Blanchini (1994) to derive a procedure to design a gain scheduling control in which the exploitation of the parameter measurement is allowed. Therefore, discrete time LPV systems are very important for the gain scheduling design, and the modeling, validation or identification play a crucial role in the overall design.

Model predictive control (MPC) can be an effective tool to design a gain scheduling policy for LPV systems. In fact, bounds in parameter variations can be explicitly considered in the optimization step of MPC, guaranteeing stability, robustness and performances. There exist several receding-horizon control schemes like Lee and Yu (1997), Kothare, Balakrishnan, and Morari (1996), Chisci, Falugi, and Zappa (2003), and Falugi, Giarré, Chisci, and Zappa (2001) which successfully address the issue of stability and constraint satisfaction for uncertain systems.

Accordingly to Chisci et al. (2003), we exploit a scheduling procedure that exhibits some of the performance properties of standard gain scheduling, but providing stability guarantees and constraints satisfaction between the scheduling points by means of robust invariant sets computation. The proposed design approach differs from traditional gain-scheduling in several aspects (Shamma & Xiong, 1999): (i) linearization errors are taken into account as linear state-dependence disturbances; (ii) input and states constraint are specified in the design procedure; (iii) rate of transitions among operating regions are explicitly considered.

1.2. Identification of LPV systems

We are considering techniques for the identification of nonlinear systems linearized along a varying trajectory. As a *model* we consider the class of discrete-time linear parameter-varying (LPV) models as in (1). The recent literature on LPV models identification started with (Nemani, Ravikanth, & Bamieh, 1995). This was, to the best of our knowledge, the first attempt to solve the problem of identifying LPV systems, considering state measurements and one scalar time-varying parameter. The problem was shown to be equivalent to a linear regression, and some conservative conditions for persistency of excitation were given. More closely related to our approach is the one of Lee and Poolla (1999), in which the problem of identifying a discrete input/output LPV model $y_k = P(p, \theta)u_k + v_k$ is addressed (p being the measured time-varying parameter) and recasted as a nonlinear programming. In Previdi and Lovera (1999), LPV models are identified with an hybrid linear/nonlinear procedure. The nonlinear part is identified through neural network and the linear parametric part trough Least Square (LS) algorithm. Finally, the approach in Mazzaro, Movsichoff, and Sanchez Pena, 1999 is to consider a robust identification via worst-case identification. The considered technique can also be improved with an iterative procedure that implement validation of the identified controlled model. An (in)validation scheme for LPV models has been presented in Sznaier, Mazzaro, and Inanc (2000).

In order to identify an input–output LPV model via linear regression with least mean square (LMS) and recursive least square (RLS) algorithms we adopt the solution developed in Bamieh and Giarré (1999, 2000, 2002).

2. The stall and surge control problem

Rotating stall and surge are critical operative conditions that limit the stability region at low mass flow in a compressor map. We observe several physical implications as a rapid heating of the blades, the increase of the compressor exit temperature, additional periodic loads, blade vibrations and fatigue that may lead to material durability reduction and severe damages to the machine. A formal distinction between the two phenomena classifies the rotating stall as the consequence of a disturbed circumferential flow pattern with the local region of stalled flow propagating around the compressor annulus at a fraction of rotor speed. The average flow is steady in time yet with a circumferentially nonuniform mass deficit as shown in Fig. 1.

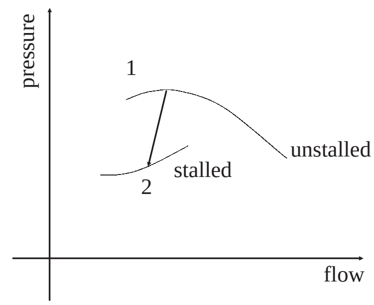

Fig. 1. Compressor map with stalled flow characteristic.

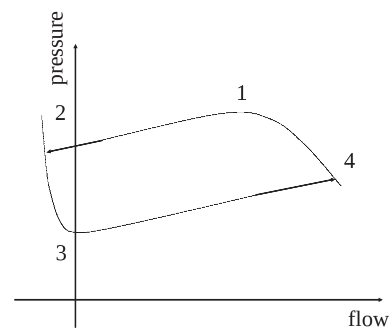

Fig. 2. Compressor map with deep Surge cycle.

On the other hand, surge is the axisymmetrical oscillation of mass flow and pressure that affects the whole compression system and causes limit cycles in the compressor characteristic. The flow is now unsteady but circumferentially uniform, as depicted in Fig. 2.

The literature presents either open loop control strategies known specifically as *surge avoidance* or closed loop strategies called *surge detection and avoidance* or *active surge control*. As an example of surge avoidance, consider Fig. 3 where two different bleed flows for constant speed/pressure prevent operations near a so-called *stall/surge margin*. Such an avoidance scheme inevitably increases energy consumption and reduces system effectiveness and efficiency.

An additional consideration is that the system response to variable load lines or during transients must also be confined within a limited region as evident from Fig. 4.

Active surge control tries to overcome the mentioned drawbacks by enhancing significantly the region of stable operations as depicted in Fig. 5.

As a matter of fact difficulties arise in deriving accurate and reliable models and integrating compressor controls with engine controls. The compression system is depicted in Fig. 6.

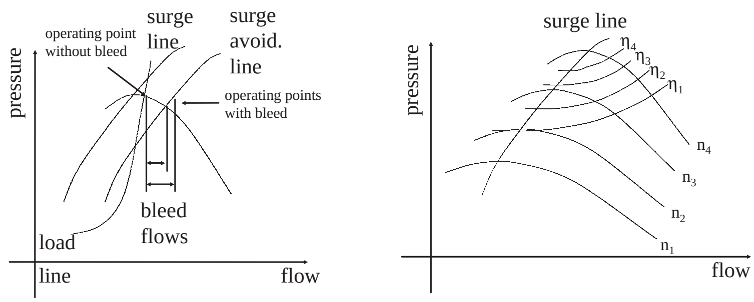

Fig. 3. Compressor map with effect of bleed and with efficiency contours.

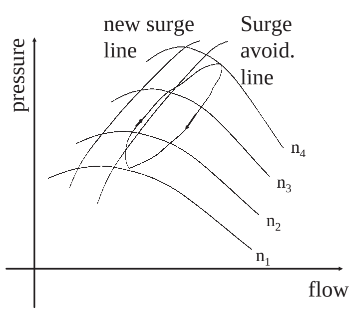

Fig. 4. Transients in aeroengines: feasible operating region restriction during acceleration/deceleration.

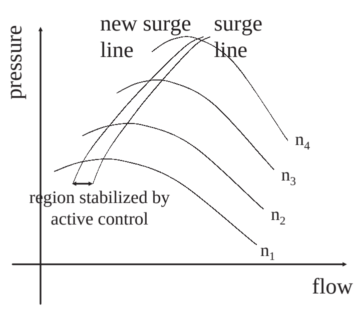

Fig. 5. Compressor map with effect of active control.

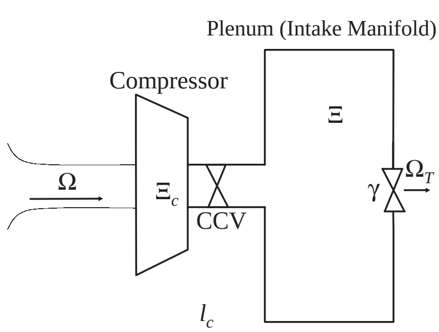

Fig. 6. Compression system.

3. Problem formulation

In this section, we introduce the simplified Moore and Greitzer model and state the problem of interest. The simplified Moore and Greitzer model, obtained from a PDE, captures the essential features of the rotating stall and surge phenomena. This is a standard model in the literature and has been successfully employed both in theoretical as well as in practical studies (Jager, 1995, Gravdahl & Egeland, 1999).

3.1. The simplified Moore and Greitzer model

We consider a low pressure rise compressor, operating at constant speed and in an essentially incompressible flow (see, e.g., Fig. 1). The simplified Moore and Greitzer model is a set of three closely coupled nonlinear differential equations in the mass flow, Ω, the pressure rise, Ξ, and the amplitude square of the first harmonic mode of the rotating stall disturbance, R. By using the single term Galerkin's Method for nonlinear mechanics (Moore & Greitzer, 1986), we have

$$\dot{\Omega} = \frac{1}{l_c}(-\Xi + \Xi_C(\Omega) - 3R(\Omega - 1)),$$

$$\dot{\Xi} = \frac{1}{4 l_c B^2}(\Omega - \Omega_T),$$

$$\dot{R} = 3\mu R(2\Omega - \Omega^2 - R), \tag{2}$$

where the time-dependence is omitted for sake of simplicity. The model input is the throttle mass flow, Ω_T, which is governed by the throttle area, γ, according to the following relation:

$$\Xi = \frac{1}{\gamma^2}\Omega_T^2. \tag{3}$$

The parameter B is the Greitzer stability parameter, which determines whether a given compressor is more likely to enter surge or rotating stall (Humbert & Krener, 1998). In this work we are interested in small to medium ranges of B. The parameter l_c is the length of compressor and duct. Similarly to Tu and Shamma (1998), here and in the rest of the paper we set $B = 0.3$, $l_c = 1$.

We also assume that the steady-state characteristic of the compression system, $\Xi_C(\Omega)$, is linearly affected by a periodical uncertainty, $\Delta\Xi_C(\Omega)$, which takes the form:

$$\Xi_C(\Omega) = \Xi_C^0(\Omega) + \Delta\Xi_C(\Omega). \tag{4}$$

In the above equation, the function $\Xi_C^0(\Omega)$ is the nominal behavior, describing the single symmetrical flow compressor map of cubic polynomial form

$$\Xi_C^0(\Omega) = 2 + \frac{3}{2}\,\Omega^2 - \frac{1}{2}\,\Omega^3. \tag{5}$$

The objective of rotating stall and surge control is to achieve the maximum compression efficiency and effectiveness while eliminating stall and stabilizing the operating point at peak pressure rise or peak compressor performance. Humbert and Krener (1998) state the importance of considering all the three state variables for feedback. This shows the advantage of eliminating the deep hysteresis associated to the primary bifurcation into stall. Then, we can state the following problem.

Problem 1 (*Surge and stall control*). *Consider the simplified Moore and Greitzer model* (2). *Assuming that all state variables are available for feedback, design a gain scheduling controller to avoid surge and rotating stall.*

Our solution approach to Problem 1 consists in the following two stages: (1) we identify and validate the LPV model from numerical simulations of the Moore and Greitzer model. The identification is performed in closed loop. The loop is closed with an initial known fixed controller designed on a priori information on the model, but open loop subsystem is identified. (2) Based on the identified LPV model, we design a scheduling controller.

4. Preliminary results

In this section, we revisit some preliminary results on LPV model identification and robust gain scheduling control of the authors: Bamieh and Giarré (1999, 2002), Falugi et al. (2001), and Chisci et al. (2003).

4.1. Identification

Our special class of discrete-time LPV models is parameterized as follows:

$$A(\delta,p)y(k) = B(\delta,p)u(k), \tag{6}$$

where y is the output and u is the input, δ is the delay operator $(\delta y)(k) := y(k-1)$, and

$$B(\delta,p) := b_0(p) + b_1(p)\delta + \cdots + b_{n_b}(p)\delta^{n_b},$$

$$A(\delta,p) := 1 + a_1(p)\delta + \cdots + a_{n_a}(p)\delta^{n_a}, \tag{7}$$

$n = n_a + n_b + 1$ is the number of parametric functions to be identified. Moreover, we assume that the varying parameter p is a function of discrete time ($p = p(k)$). We assume that the functions $\{a_i\}$, $\{b_j\}$ in (7) are polynomials in p of order $N-1$, i.e.

$$a_i(p) = a_i^1 + a_i^2 p + \cdots + a_i^N p^{N-1},$$

$$b_j(p) = b_j^1 + b_j^2 p + \cdots + b_j^N p^{N-1}. \tag{8}$$

Thus any particular model in our class is completely characterized by the real numbers $\{a_i^k\}$ and $\{b_j^l\}$. We now observe that the input–output model in (6), together with (8) and (7) can be put into a nice linear regression form. It is easy to verify that (see Bamieh & Giarré, 2002) the model (7)-(8) can now be rewritten as

$$y_k = \langle \Theta, \Psi_k \rangle, \tag{9}$$

where the inner product is defined as follows: given any two matrices A, B of equal dimensions, their inner product is

$$\langle A, B \rangle := \mathrm{trace}(A^* B) = \mathrm{trace}(BA^*),$$

where A^* means complex conjugate transpose of the matrix A. For given integers (n,m), this inner product makes $\Re^{n\times m}$ (the space of $n \times m$) matrices into an inner product space; the $n \times N$ matrix Θ which contains all the coefficients to be identified is defined as

$$\Theta := \begin{bmatrix} a_1^1 \ldots a_1^N \\ a_2^1 \ldots a_2^N \\ \vdots\vdots\vdots \\ a_{na}^1 \ldots a_{na}^N \\ b_0^1 \ldots b_0^N \\ \vdots\vdots\vdots \\ b_{nb}^1 \ldots b_{nb}^N \end{bmatrix}. \tag{10}$$

and the *extended regressor* Ψ_k is defined by

$$\Psi_k := \phi_k \pi_k$$

$$:= \begin{bmatrix} -y_{k-1} \\ \vdots \\ -y_{k-n_a} \\ u_k \\ \vdots \\ u_{k-n_b} \end{bmatrix} \begin{bmatrix} 1 & p_k & \ldots & p_k^{N-1} \end{bmatrix}, \tag{11}$$

where, for notational simplicity, we use a subscript for the time index (e.g. $p_k := p(k)$).

The advantage of rewriting the model in the inner product form (9) is that the usual RLS and LMS algorithms can now be easily generalized to this case. Then it is possible to consider least squares type identification problems for LPV systems with polynomial dependence on the parameters. By representing the regression as a matrix inner product, in Bamieh and

Giarré (1999, 2002) a simple representation of the RLS and LMS algorithms, and conditions for persistency of excitation have been derived. We report hereafter the LMS algorithm:

LMS Algorithm. *For the model in* (6), *the Least Mean Square identification algorithm is given by*

$$\Psi_k = \phi_k \pi_k, \tag{12a}$$

$$\varepsilon_k = y_k - \langle \hat{\Theta}_k, \Psi_k \rangle = y_k - \mathrm{trace}(\hat{\Theta}_k^T \Psi_k), \tag{12b}$$

$$\hat{\Theta}_{k+1} = \hat{\Theta}_k + \alpha \varepsilon_k \Psi_k. \tag{12c}$$

4.2. Robust gain scheduling predictive control

Following Shamma and Xiong (1999) we consider the discrete-time LPV system

$$x(k+1) \in \mathscr{F}(p(k)) \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} + Dw(k),$$
$$x(0) = x_0, \ \forall w(k) \in W,$$
$$p(k+1) \in Q(p(k)), \quad p(0) = p_0, \tag{13}$$

where $x(k) \in \mathbb{R}^n$ is the state vector and $u(k) \in \mathbb{R}^m$ the control input, $p(k)$ is a time-varying parameter belonging to a discrete-set $P = \{p_1, \ldots, p_l\}$ and evolving according to a set-valued map $Q: P \rightsquigarrow P$, $w(k)$ is a disturbance and W is a compact and convex set. Finally, the map $\mathscr{F}: P \rightsquigarrow \mathbb{R}^{n \times (n+m)}$ represents additional uncertainty in the system dynamics.

Assumption 1. For any $p_j \in P$, $\mathscr{F}(p_j)$ is a closed convex polytope P_j, i.e:

$$\begin{aligned} &x(k+1) = A(k)x(k) + B(k)u(k) + Dw(k) \\ &\text{for some } [A(k), B(k)] \in P_j. \end{aligned} \tag{14}$$

Moreover system (13) is subject to point wise-in-time control and state constraints

$$u(k) \in \mathscr{U}, x(k) \in \mathscr{X}, \quad \forall k \geqslant 0 \tag{15}$$

for some appropriate polyhedra $\mathscr{U} \subset \mathbb{R}^m$, $\mathscr{X} \subset \mathbb{R}^n$ containing the origin as an interior point.

Assumption 2. For each value p_j of the time-varying parameter there exists a linear feedback control law $u(k) = F_j x(k)$ such that the closed loop polytopic system

$$x(k+1) \in \mathscr{F}(p_j) \begin{bmatrix} x(k) \\ F_j x(k) \end{bmatrix} \tag{16}$$

is absolutely asymptotically stable.

In this respect, we investigate: (1) if the linear gain scheduling control law $u(k) = F_j x(k)$ stabilizes the LPV system and (2) how to design a non-linear state-feedback regulator

$$u(k) = g(x(k), p(k)), \tag{17}$$

which improves the performance of the linear one. Notice that, because of constraints (15), a simple linear feedback strategy may result in a small stability region. As a remedy, the nonlinear feedback (17) could, in principle, significantly enlarge the stability domain.

Let Σ_0 denote the set of initial extended states $s(0) = [x'(0), p'(0)]' \in \mathbb{R}^n \times P$ for which the plant state is ultimately bounded under the gain-scheduled linear feedback (16) without violating the constraints. Further, consider the LPV system

$$x(k+1) \in \mathscr{F}(p(k)) \begin{bmatrix} x(k) \\ F(p(k))x(k) + c(k) \end{bmatrix} + Dw(k),$$
$$p(k+1) \in Q(p(k)),$$
$$F(p) = F_j \quad \text{if } \ p = p_j$$

for which the actual input sequence is computed as a sum of the gain-scheduled linear feedback (16) plus an open-loop signal $c(\cdot)$. Let $\mathscr{S}_N$ be the set of vectors $[s'(0), c'(0), ..., c'(N-1)]'$ such that $\{c(0), c(1), ..., c(N-1)\}$ steers the initial state s_0 to Σ_0 in N steps. For the calculation of the invariant polytopes S_N, see Chisci et al. (2003) where the following predictive control algorithm has been proposed.

Parameter varying - predictive control (PV-PC) Algorithm. *At each sample time* k, *given* $s(k) = [x(k)', p(k)']' \in \mathbb{R}^n \times P$, *find*

$$\hat{\underline{c}}(k) = \mathrm{argmin} \| \underline{c}(k) \|^2, \tag{18}$$

subject to

$$\begin{bmatrix} s(k) \\ \underline{c}(k) \end{bmatrix} \in S_N. \tag{19}$$

Then apply to the plant the control signal

$$u(k) = F_j x(k) + \hat{c}_1(k) \tag{20}$$

where $\hat{\underline{c}}(k)^T = [\hat{c}_1^T, \ldots, \hat{c}_N^T]$ *and* j *is such that* $p(k) = p_j$.

At time k, the above algorithm selects among all admissible sequences $\underline{c}(k)$ the one with minimum l_2 norm. Since S_N is a collection of convex polytopes (18)-(19) amounts to a *quadratic programming (QP)* problem. As far as stability is concerned, results can be found in Chisci et al. (2003).

5. Solution approach and main results

We highlight now the two main contributions of the paper: (i) given the well known Moore and Greitzer model (2), we identify and validate an LPV model, (ii) given the LPV model, we design a scheduling controller for the stall and surge control.

More in detail:

(i) We need to convert the Moore and Greitzer model (2) into an LPV form and identify the parameters according to Bamieh and Giarré (2002). Preliminarily, it will be necessary to locally stabilize the model (2) via a state feedback, as shown in the next subsection. The identification of the open loop subsystem, is then performed in closed loop.

(ii) The PV-PC controller consists in (i) a gain-scheduled linear feedback, designed off-line and (ii) an on line nonlinear correction. The off line linear feedback guarantees stability and constraints satisfaction between the scheduling points by means of robust invariant sets computations. This controller is a gain scheduling procedure that exhibits some of the standard performance properties and provides stability guarantees and constraints satisfaction between the scheduling points by means of robust invariant sets computation. The on-line nonlinear correction is the result of a receding horizon optimization and is used as a further degree of freedom in order to enlarge the stability domain and to improve performance.

5.1. The LPV form of the Moore and Greitzer model

According to Tu and Shamma (1998), we convert the Moore–Greitzer model in LPV form, deriving a Quasi LPV model. We shift the phase plane origin at peak pressure rise point, $\omega(k) = \Omega(k) - 2$, with balanced throttle mass flow, $\omega_T(k) = \Omega_T(k) - 2$ and choose the variable $p(k) = \omega(k)$ as the *scheduling* parameter. Now, by using the relaxation auxiliary variable $v(k)$ we get the following LPV model:

$$\mathbf{x}(k+1) = A(p(k))\mathbf{x}(k) + B_1 u(k) + B_2 d(k),$$

$$\mathbf{y}(k) = C(p(k))\mathbf{x}(k),$$

$$z(k) = C_1(p(k)), \tag{21}$$

where the state variable is

$$\mathbf{x}(k) = [v(k) \quad \xi(p(k)) - \xi_{eq}(p(k)) \quad R(k)]',$$

the input is $u(k) = \omega_T(k) - \omega_{T_{eq}}(k)$ and the term $d(k) = 0.75\sin(2p(k)) + 0.25\sin(14p(k))$ is a disturbance, which takes into account the compressor characteristic uncertainties. In the above model the dynamic matrix is given by

$$A(p(k), \rho(k)) = \begin{bmatrix} 1 & -\frac{T}{l_c} & -\frac{3T}{l_c}(p+1) \\ 0 & 1 - \frac{T}{l_c}(3p + \frac{3}{2}p^2) & -\frac{3T}{l_c}(3p + \frac{3}{2}p^2)(p+1) \\ 0 & 0 & 1 - 3\mu T(2p + p^2) - 0.9\mu T\rho \end{bmatrix}, \tag{22}$$

where $\rho = \rho(k)$ is a disturbance taking into account some nonlinear terms, and $p = p(k)$ is the scheduling parameter. We also have,

$$B_1 = \begin{bmatrix} 0 \\ -\frac{T}{4 l_c B^2} \\ 0 \end{bmatrix}, \tag{23}$$

$$B_2 = \begin{bmatrix} 0.2T \\ 0 \\ 0 \end{bmatrix}, \tag{24}$$

$$C_1(p(k)) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{T}{l_c} & -\frac{3T}{l_c}(p(k)+1) \\ 0 & 0 & 1 \end{bmatrix}. \tag{25}$$

and $C = I$, $D = 0$. The sampling time T is equal to 0.04 s.

In order to identify the model, we must first stabilize the system in a neighborhood of the shifted origin. To see the instability region in open loop, we simulate, under the condition of surge ($R = 0$, $\gamma = 0.85$) the transient response from an initial state, $x(0) = [-0.15 \quad 0 \quad 0]$. The plot of the simulated data displayed in Fig. 7 shows the presence of a limit cycle.

Remark. We note that the simplified Moore and Greitzer cannot be exponentially stabilized by means of a $\mathscr{C}^1$ static controller. To see this fact consider the linearized dynamics and notice that there exists an uncontrollable mode which is marginally stable and not asymptotically convergent to zero. This is the mode associated to the dynamics of R. Due to this inconvenience, the computation of the invariant sets theoretically would requires an infinite number of recursions. In order to overcome this problem, we set $\rho(k) = 0.95$

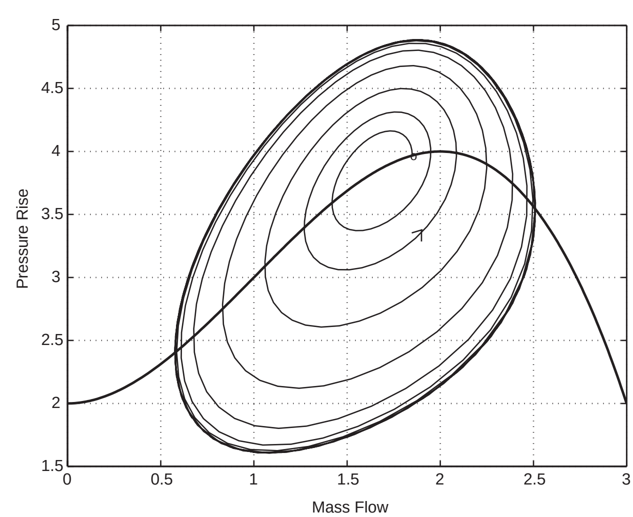

Fig. 7. Steady axisymmetric compressor characteristic: transient response with classic surge cycle.

instead of $|\rho(k)| \leqslant 1$ in the design procedure. This is equivalent to assume that the dynamics of R is exponentially stable, so we can compute an invariant region for the LPV model. Even though we can not guarantee that the original model is regulated at the desired equilibrium point, it is possible to analyze the system under the designed control action. It is useful to consider the simplified Moore and Greitzer as a cascade of two subsystems. The first subsystem involves the dynamics on Ω and Ξ, and considers the influence of R, with $0 \leqslant R \leqslant 0.3$, as a disturbance. The second subsystem involves the dynamics on R subject to the input Ω. By choosing $V = R^2/2$ as the Lyapunov function it iseasy to show that, for $R \geqslant 0$, the system is ISS (Input to State Stable) as

$$\dot{V}(R, \Omega) \leqslant -\gamma(|R|) + \alpha(|\Omega - 2|) \quad \forall R \geqslant 0, \Omega$$

with $\gamma(|R|) = 3 * \mu R^2|R|/2$ and $\alpha(|\Omega - 2|) = 12\mu((\Omega - 2)^2 + 2|\Omega - 2|)^3$ which are $\mathscr{K}_\infty$ functions. Since the first block turns to be robustly locally asymptotically stable under a properly designed feedback it is possible to conclude (see, e.g., the cascade theorem, Teel & Sontag, 1995) that the overall system is locally asymptotically stable for $R \geqslant 0$.

5.2. Identification of the LPV model: a numerical example

In order to identify the model, the system has to be first stabilized in a neighborhood of the shifted origin with a feedback control of the form $u(t) = [k_1\ k_2\ k_3]\mathbf{x}(t) + \underline{u}(t)$. Considering a finite set of possible values for the parameters, we obtain a feedback control law solving an LQ optimal control corresponding to each value of the parameter set. Measuring the parameter value, the feedback is chosen and applied to the nonlinear Moore and Greitzer model (2). Then, collecting the data $\{u_k, y_k, p_k\}$ from the simulated closed loop nonlinear system, we identify the open loop LPV model.

Note that the state space LPV model given by (22), (24), (25) can be easily put in the fractional representation needed for the identification. Recalling that the system is simulated under the operating condition of surge ($R = 0$, i.e. the third state variable is equal to zero), we identify the model corresponding to the first two outputs (the first two state variables).

According to (7), the open loop LPV model is

$$A(\delta, p)\mathbf{y}(k) = \mathbf{B}(\delta, p)u(k) + \mathbf{B_d}(\delta, p)d(k),$$

where the matrix

$$A(\delta, p) = 1 + A_1(p)\delta^{-1} + A_2(p)\delta^{-2}$$

with coefficients varying according to the polynomials $A_i(p) = a_i^1 + a_i^2 p + a_i^3 p^2$ for $i = 1, 2$. The matrix

$$\mathbf{B}(\delta, p) = \begin{bmatrix} B_1(\delta, p) \\ B_2(\delta, p) \\ 0 \end{bmatrix},$$

where $B_1(\delta, p) = \beta_{12}(p)\delta^{-2} + \beta_{13}(p)\delta^{-3}$, $B_2(\delta, p) = \beta_{21}(p)\delta^{-1} + \beta_{22}(p)\delta^{-2} + \beta_{23}(p)\delta^{-3}$ and

$$\mathbf{B_d}(\delta, p) = \begin{bmatrix} \beta_{d11}\delta + \beta_{d12}\delta^{-2} + \beta_{d13}\delta^{-3} \\ 0 \\ 0 \end{bmatrix}.$$

Given the above LPV form, we identify the parameters via the LMS algorithm (12). To guarantee a rapid convergence of the algorithm we choose as step size $\alpha = 20$ in (12c). Particular accuracy is required in choosing the input $\underline{u}$ and the variations of the scheduling parameters p thus to satisfy the persistency conditions (see, e.g., Bamieh & Giarré, 2002). Note that the identification scheme requires the knowledge of the i/o data from the real system, and the measured parameter varying p. The system needs to be controlled so that the shifted-variable corresponding to the parameter variable follows a reference $\bar{p}$. In particular, the reference $\bar{p}_k$ is chosen as 0.15 sin($6k$), since 6 is not a rational multiple of the transcendental number π. Therefore, sin($6k$) is not a periodic function of k, however its range of values is dense in $[-0.15, 0.15]$. Note that this is the interval of operating condition for the measured parameter of the nonlinear system. Moreover, it can be shown that given any neighborhood of a point in $[-0.15, 0.15]$, sin($6k$) will revisit this neighborhood at a rate of order $6k$. Moreover $\underline{u}$ is chosen as a uniformly distributed signal in the range $[-0.2, 0.2]$. Note that an additive uniformly distributed noise affecting the output, independent of $\underline{u}$ is also considered.

From the identified parameters, we easily get the entries of the matrices A, B_1, B_2 and C_1 in Eqs. (22), (24), (25).

The length of simulation is $N = 4000$ steps. Now, given a value of p and the corresponding feedback gain, the identification, performed on line, returns the following final estimates for the matrices $\hat{A}_i, i = 1, 2$:

$$\begin{bmatrix} \hat{A}_1(p) \\ \hat{A}_2(p) \end{bmatrix} = \begin{bmatrix} -1.99 + 0.11p + 0.06p^2 \\ 0.99p - 0.12p - 0.06p^2 \end{bmatrix}.$$

Fig. 8 displays the time plot of the estimates of the coefficients of $\hat{A}(p)$.

Similarly, the final estimate of the coefficients of polynomial $B_1(p)$ and $B_2(p)$ are:

$$\hat{B}_1(p) = \begin{bmatrix} 0.0004 + 0.0002p \\ 0.0031 + 0.0005p \\ 0.0023 + 0.0008p \end{bmatrix},$$

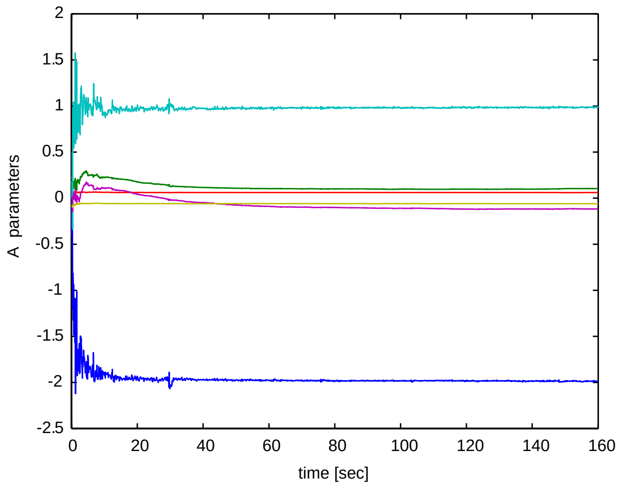

Fig. 8. LMS algorithm: time plot of the $A(p)$-parameters estimation.

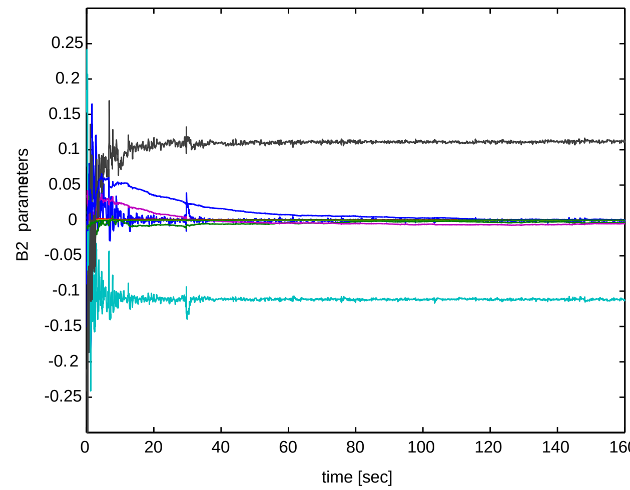

Fig. 10. LMS algorithm: time plot of the $B_2(p)$-parameters estimation.

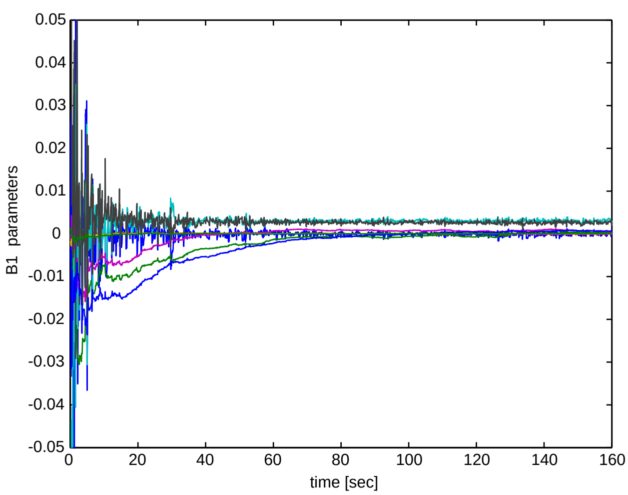

Fig. 9. LMS algorithm: time plot of the $B_1(p)$-parameters estimation.

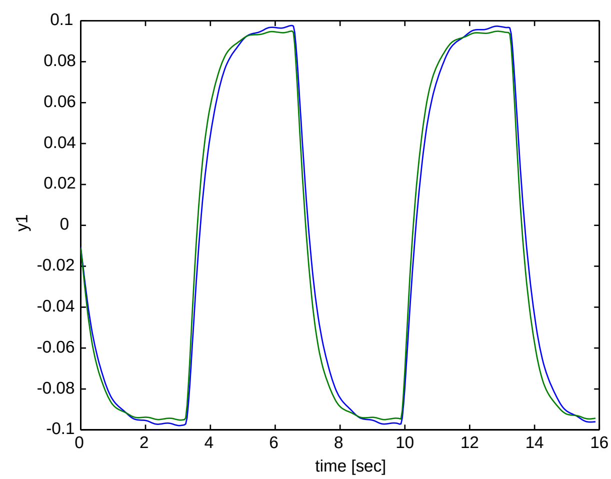

Fig. 11. Time response plot of output y_1 of the nonlinear system and the identified LPV model.

$$\hat{B}_2(p) = \begin{bmatrix} 0.0006 - 0.003p \\ -0.112 - 0.0047p \\ 0.112 + 0.001p \end{bmatrix},$$

Figs. 9 and 10 display the time plot of the estimates of the coefficients of, respectively, $B_1(p)$ and $B_2(p)$.

Now, in order to validate the identification results, we simulate the response to a square wave input of the LPV model and compare the outputs with the ones obtained from the Moore and Greitzer model. Figs. 11 and, 12 show, respectively, the evolution of the outputs y_1 and y_2 over a length of 400 steps.

5.3. Controller design

The aim is now to design a robust overall stabilizing controller. In particular, on the basis of the identified LPV model, we synthesize a gain scheduling controller achieving constrained regulation. To do this, we follow the same line of reasoning as in Falugi et al. (2001). Consider a set of 16 possible values for the scheduling parameters p_j as follows: $\{-0.15, -0.13, -0.11, \ldots -0.01, 0.01, 0.03, \ldots, 0.15\}$. Now, for each value of the scheduling parameter, we design an LQ optimal feedback control of the form (16). Let F_j be the feedback corresponding to $(p_j + p_{j+1})/2$. The feedback F_j applies to any measured value $p(k) \in [p_j\ p_{j+1}]$ and it guarantees stability (see, e.g., Assumption 2 in Section 4.2) under the following parameter variation $p(k+1) \in Q(p(k))$, with dynamics:

$$Q(p) = \begin{cases} \{p_1, p_2\}, & p = p_1, \\ \{p_{j-1}, p_j, p_{j+1}\}, & p \in \{p_2, p_3, \ldots, p_{l-1}\}, \\ \{p_{l-1}, p_l\}, & p = p_l. \end{cases}$$

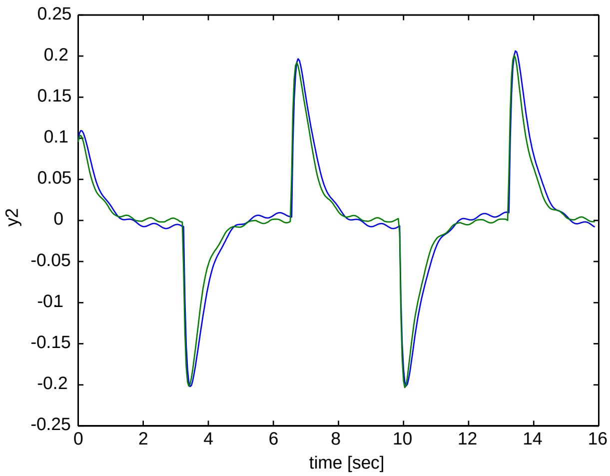

Fig. 12. Time response plot of output y_2 of the nonlinear system and the identified LPV model.

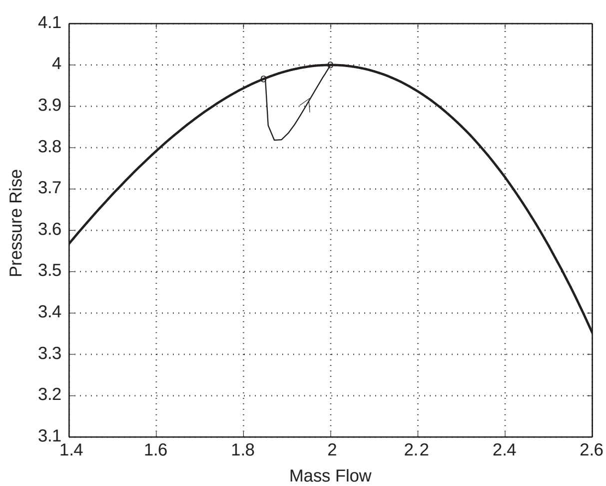

Fig. 13. Steady axisymmetric compressor characteristic: controlled transient response.

Given the above feedback law, we compute the set S_N required by the PV-PC control algorithm (18) and (19). Moreover, the invariant set of the PV-PC control has been computed by considering all the extended states which can be driven in N steps to Σ_0 by exploiting a control sequence $u(k) = g(x(k), p(k))$ (see for details, Chisci et al., 2003) which explicitly takes into account the state evolution.

Now, the aim is to test the controller capability of robustly stabilizing stall and surge. For doing so, we first simulate the response of the Moore and Greitzer model under instability conditions and shown in Fig. 7. Choosing a control horizon length $N = 2$, we apply the PV-PC control algorithm (18) and (19) to the Moore and Greitzer model. The simulation results in Fig. 13 show that the PV-PC controller prevents from the insurgence of limit cycles and stabilize the system at the peak pressure rise point.

For the second part, we simulate the insurgence of stall by choosing an initial state $x(0) = [-0.15, \quad 0, \quad 0.3]'$. Figs. 14, 15 and 16 plot the time evolution of the state variables of the Moore and Greitzer model in closed loop for increasing control horizon length $N = 2, 3, 4$. The horizon is greater than 1, because the above initial conditions are not feasible for $N = 1$. Note that the first harmonic of the rotating stall R in Fig. 16 converges to zero, which means that the rotating stall is neutralized by the gain scheduling feedback. Furthermore, observe that the range of variations of the state variables satisfy the constraints. It is also evident that there is no significant improvement in performance by extending the control horizon length beyond $N = 2$.

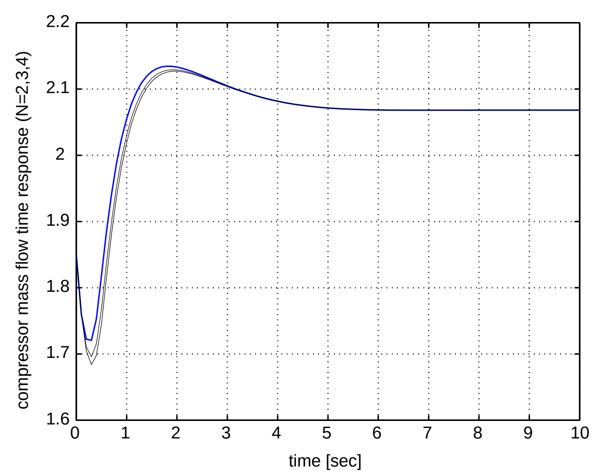

Fig. 14. Compressor mass flow time response ($N = 2, 3, 4$).

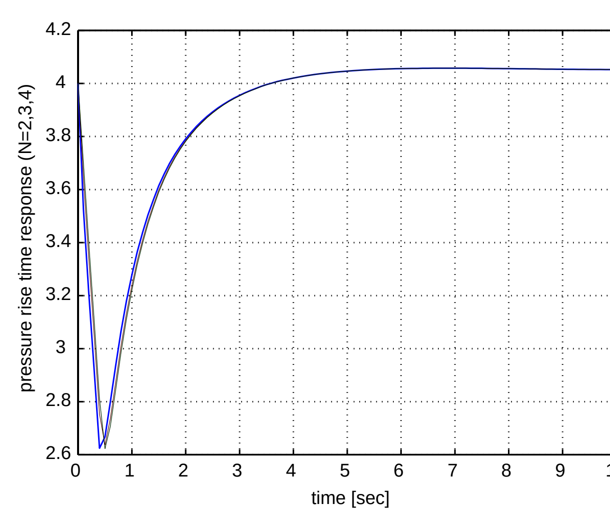

Fig. 15. Pressure rise time response ($N = 2, 3, 4$).

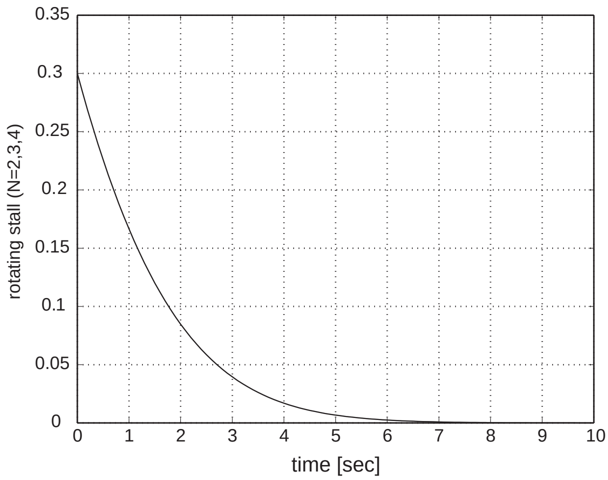

Fig. 16. Rotating stall ($N = 2, 3, 4$).

6. Conclusions

In order to design a gain scheduling controller it is crucial to have a simple LPV model of the nonlinear system. Identification of an LPV embedding for the Moore and Greitzer model is performed in order to later design a controller for the rotating stall and surge control of jet engines.The adopted PV-PC controller consists in (i) a gain-scheduled linear feedback, designed off-line and (ii) an on-line nonlinear correction. This correction is the result of a receding horizon optimization based on invariant sets theory and it allows the enlargement of the stability basin of attraction. For the identification, we have considered Least Squares type algorithms for LPV systems with polynomial dependence on the parameters, without the requirements of slow variations amongst set points. The simulation results have shown that the PV-PC controller robustly stabilizes stall and surge.

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