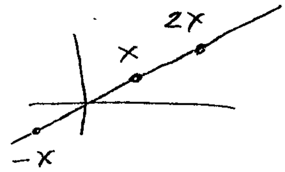
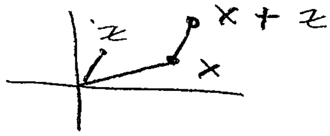
S Modica 8 Feb 2011

$\mathbb{R}^n$ e funzioni lineari e Concave; Prodotto Scalare

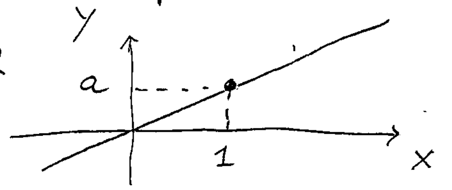
- $\mathbb{R}^n \ni x = (x_1, \dots, x_n)$
  - prodotto tx, $t \in \mathbb{R}$
    $tx \equiv (tx_1, \dots, tx_n)$



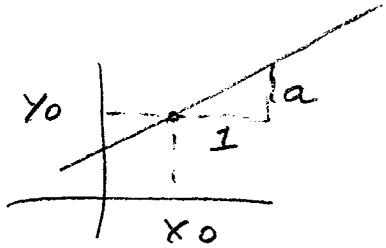
  - Somma $x + z$, $x, z \in \mathbb{R}^n$
    $x+y \equiv (x_1+z_1, \dots, x_n+z_n)$



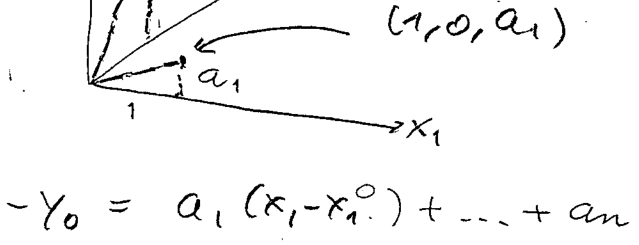
- Retta in $\mathbb{R}^2$
  - $y = ax$ Grafico è l'insieme dei punti
    $$(x, ax) = x(1,a), \; x \in \mathbb{R}$$
    'generato da' $(1,a)$.



  - $y - y_0 = a(x - x_0)$
    Stessa cosa a partire da (y_0, x_0)
    grafico: insieme punti
    $$(x_0, y_0) + x(1,a), \; x \in \mathbb{R}.$$



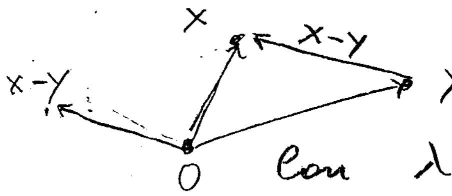
- Funzione lineare in $\mathbb{R}^{n+1}$
  - $y = a_1x_1 + a_2x_2 + \dots + a_nx_n$
    Grafico: insieme punti in $\mathbb{R}^{n+1}$
    $$(x_1, \dots, x_n, a_1x_1 + \dots + a_nx_n), \; x_1 \dots, x_n \in \mathbb{R}$$
    $$= x_1(1, 0, \dots, 0, a_1) + \dots + x_n(0, 0, \dots, 1, a_n)$$
    generato da $(1, \dots, 0, a_1), \dots (0, \dots, 1, a_n)$



si chiama (Iper) Piano

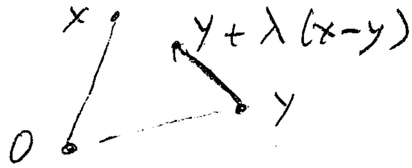
  - $y - y_0 = a_1(x_1 - x_1^0) + \dots + a_n(x_n - x_n^0)$
    Stessa cosa a partire da (x^0, y_0).

- Combinazioni convesse



Per 'vedere' $x-y$ scrivilo (e percorrilo) come $-y+x$, partendo da y.

Con $\lambda \in (0,1)$, $\lambda x + (1-\lambda) y$ si chiama combinazione convessa di x ed y. È $= y + \lambda(x-y)$

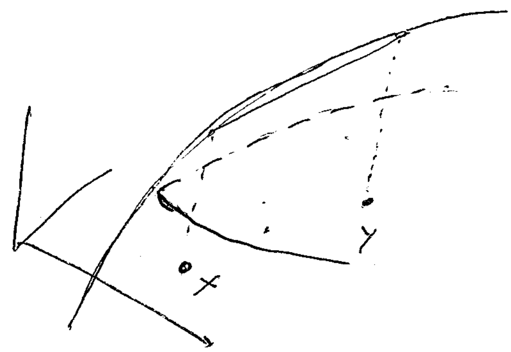


- Funzioni concave (def)

$f: \mathbb{R}^n \to \mathbb{R}$ è concava se $\forall x, y \ \forall \lambda \in (0,1)$

$$f(\lambda x + (1-\lambda) y) \geq \lambda f(x) + (1-\lambda) f(y)$$

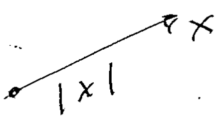
convessa se $\leq$.

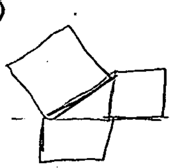


Oss. Se f è concava lo è in ogni variabile (prendi X con tutte coordinate uguali tranne una), quindi in part. $\partial^2 f / \partial x_k^2 \leq 0$.

UN PO' DI GEOMETRIA
(SE TI VA)

o Lunghezza

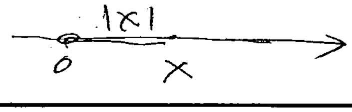




PITAGORA

$$|x| = \left(\Sigma_i x_i^2\right)^{1/2} \qquad x \in \mathbb{R}^n$$

Nota per $n=1$ è il val. assoluto (cioè la lunghezza



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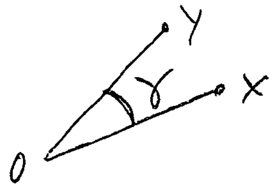
- **Prodotto scalare**

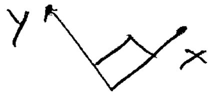
$x, y \in \mathbb{R}^n \qquad x \cdot y = \sum x_i y_i.$

nota $x \cdot x = |x|^2$



- Se $n = 2$

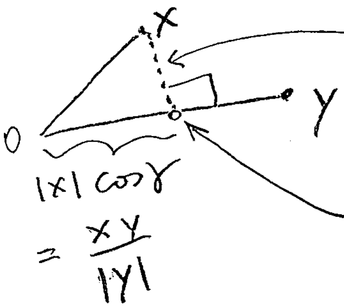
(1) $x \cdot y = x_1 y_1 + x_2 y_2 = |x| \cdot |y| \cos \gamma$



(2) $x \cdot y = 0 \iff \gamma = \pi/2 \iff x \perp y$

(3) $|x \cdot y| \leq |x| \cdot |y|$ (perché $|\cos \gamma| \leq 1$)

(4)



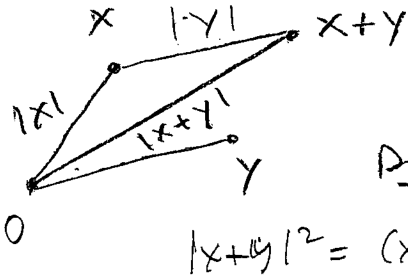
$\left(x - \frac{xy}{|y|^2} y\right) \cdot y = 0$

$\frac{xy}{|y|^2} y$

- Di nuovo $\mathbb{R}^n$.
  - La (3) vale ancora (Cauchy-Schwartz)

Dim (Wikipedia): la parabola non negativa in z $(x_1 z + y_1)^2 + \dots + (x_n z + y_n)^2$ ha discriminante non positivo $4|xy|^2 - 4|x|^2|y|^2$ $\square$

- Vale la Disuguaglianza Triangolare



$|x + y| \leq |x| + |y|.$

Dim (Wikipedia, Triangle Inequality)

$$|x+y|^2 = (x+y)\cdot(x+y) = |x|^2 + 2x \cdot y + |y|^2 \underset{(C-S)}{\leq} |x|^2 + 2|x|\cdot|y| + |y|^2 = (|x|+|y|)^2. \quad \square$$